UNIVERSITY OF TORONTO

Faculty of Arts and Science

APRIL/MAY EXAMINATIONS 1997

PHY 357S

Duration - 3 hours

This exam has 4 pages.

Allowed aids: Calculator and 1 letter-sized (21.5 by 30 cm) double-sided hand-written aid sheet.

Do 4 of 5 questions; each question has equal weight.

Each part of a multipart question is of equal weight.

If you do all 5 questions, your best 4 marks will be used.

Show your work and reasoning; part marks will be given.

***** Some of the following numerical constants may be useful:

pi	= 3.14159265358979323…	
speed of light:	c = 299792458 m s-1	
Planck's constant, reduced:	$\hbar$ = 6.58212×10^{-22} MeV s	
conversion constants:	$\hbar$ c =197.327 MeV fm	$(\hbar\ c)^2$ =0.389379 GeV^2 mb
conversion factors:	1 eV = 1.602177×10^{-19} J	1 eV/c2 = 1.782662×10^{-36}kg
	1 barn = 10^{-24} cm^2	
unified atomic mass unit		u = 931.494 MeV/c^2
Avogadro's constant		N_A = 6.022137×10^{23} mol^{-1}
electron charge:		e = 1.602×10^{-19} C
electron magnetic moment:		μ_e = 2.9665×10^{-7} eV^{-1}
Boltzmann constant		k = 8.617×10^{-5} eV/Kelvin
fine structure constant		α= 1/137.0360
strong coupling constant		$\alpha_S(M_Z)$= 0.118±0.003
Cabibbo angle		$\sin\theta_c \cong V_{us}$ = 0.22
weak mixing angle		$\sin^2\hat{\theta}(M_Z)$ = 0.2315
standard grav. accel., sea level:		g = 9.8 m/s^2
Hubble constant		H_0=0.1 Gyr^{-1}
neutral kaon mass difference:		$m_{K_L} - m_{K_S}$ = 3.510±0.018 μeV
Fermi weak coupling constant	G_F = 1.166×10^{-5} GeV^{-2}	
Newton's (gravitational) constant	G_N = 6.673×10^{-11}m^{3}kg^{-1}s^{-2} = 6.707×10^{-39} GeV^{-2}	
Z^0 branching fractions	B.R.($Z^0 \rightarrow e^+e^-$)=3.21±0.07%	B.R.($Z^0\rightarrow$hadrons) = 71±1%

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* Particle Properties *

Boson	Mass (GeV/c^2)
γ	$<3\times10^{-36}$
gluon	<0.01
$W^{\pm}$	80.22
Z^0	91.187
H^0	>58
graviton	~0

Lepton	Mass (MeV/c^2)	Mean Lifetime
ν_e	$<10^{-5}$	
e^-	0.510999	$>4.3\times10^{23}$ y
ν_μ	<0.27	
μ^-	105.658	2.197 μs
ν_τ	<10	
τ^-	1777	0.29 ps

Quarks	mass (GeV/c^2)
u	~0.005
d	~0.01
c	1.0-1.5
s	0.1-0.3
t	174±6
b	~4.5

Hadron	Quark Content	Mass (MeV/c^2)	$I(J^{PC})$
^{0}	$(u\bar{u}-d\bar{d})/\sqrt{2}$	134.97	$1(0^{-+})$
$^+$, $^-$	$u\bar{d}$, $d\bar{u}$	139.57	$1(0^-)$
K^+,K^-	$u\bar{s}$, $s\bar{u}$	493.65	$\frac{1}{2}(0^-)$
$K^0,\bar{K}^0$	$d\bar{s}$, $s\bar{d}$	497.67	$\frac{1}{2}(0^-)$
ρ^+, ρ^0, ρ^-	$u\bar{d}$, $(u\bar{u}-d\bar{d})/\sqrt{2}$, $d\bar{u}$	770	$1(1^{--})$
p	uud	938.27	$\frac{1}{2}(\frac{1}{2}^+)$
n	udd	939.57	$\frac{1}{2}(\frac{1}{2}^+)$
Λ	uds	1115.6	$0(\frac{1}{2}^+)$
Σ^0	uds	1192.6	$1(\frac{1}{2}^+)$
$\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$	ddd, udd, uud, uuu	1232	$\frac{3}{2}(\frac{3}{2}^+)$
$\bar{D}^0, D^0$	$u\bar{c}$, $c\bar{u}$	1863	$\frac{1}{2}(0^-)$
D^-,D^+	$d\bar{c}$, $c\bar{d}$	1869	$\frac{1}{2}(0^-)$
Λ_c^+	udc	2285	$0(\frac{1}{2}^+)$
B^+,B^-	$u\bar{b}$, $b\bar{u}$	5279	$\frac{1}{2}(0^-)$
Λ_b^0	udb	5640±50	$0(\frac{1}{2}^+)$

Nuclei	^1H	^2H	^3H	^3He	^4He	^{12}C	^{56}Fe	^{208}Pb
Mass (in u)	1.007825	2.0140	3.01605	3.01603	4.00260	12	55.934939	207.976627
Proton Number (Z)	1	1	1	2	2	6	28	82
Neutron Number	0	1	2	1	2	6	28	126

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Problem 1

(a) An undergraduate lab experiment observes muons passing through a 1 m thick tank of water. Estimate the average energy lost by cosmic ray muons passing through the tank.

(b) Why does the fractional energy resolution ($\delta E/E$) of calorimeters get better (i.e. smaller) with increasing energy E, while the fractional momentum resolution ($\delta p/p$) for measuring momentum using tracking detectors in a magnetic field gets worse?

Problem 2

Which of the following decays or reactions can take place? If forbidden, state by what selection rule. If allowed, indicate through which interaction the decay or reaction will proceed. (For the reactions, you may assume the beam particles have very high energy.)

(a) $\pi^0 \rightarrow \gamma\gamma$

(b) $n \rightarrow \gamma\gamma$

(c) $n \rightarrow p\, e^- \nu_e$

(d) $n \rightarrow p\, e^+ \nu_e$

(e) $e^- p \rightarrow e^- n\, \pi^+$

(f) $pp \rightarrow {}^2H\, e^+ \nu_e$

(g) $pp \rightarrow n\, n\, \pi^+ \pi^+$

(h) $pp \rightarrow \Lambda p\, \pi^+$

Problem 3

The W boson mass is 80 GeV. The fraction of W bosons which decay in the mode $W^+ \rightarrow \mu^+ \nu_\mu$ is 10%. Consider the interactions of cosmic ray electron antineutrinos with electrons at rest.

(a) Show that $\bar{\nu}_e\ e^-$ scattering is possible even if only charged weak currents exist.

(b) What energy should the electron antineutrinos have for the $\bar{\nu}_e\ e^-$ cross section to be the maximum possible?

(c) What is this maximum cross section (in barns or cm^2)? You may assume the neutrinos and electrons are unpolarized.

Problem 4

(a) Show that the helicity $\vec{\mathbf{J}} \cdot \hat{\mathbf{p}}$ is not invariant under the parity operation.

(b) Light stable nuclei often contain equal numbers of neutrons and protons, but very heavy stable nuclei always have more neutrons than protons. Why do heavy stable nuclei contain more neutrons than protons?

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Problem 5

(a) Assume billiard balls have a diameter of 5 cm and a mass of 0.15 kg. What is the maximum momentum transfer (**q**) possible in the collision of a moving billiard ball (velocity 1 m/s) with an identical billiard ball at rest? (You should ignore friction, and assume the stationary target ball is free to move.)

(b) In the Born approximation, the scattering amplitude can be written as

$$f(\vec{\mathbf{q}}) = -\frac{m}{2\pi\hbar^2}\int V(\vec{\mathbf{x}})\; e^{i\vec{\mathbf{q}}\cdot\vec{\mathbf{x}}/\hbar}\; d^3x$$

where V(**x**) is the scattering potential. Derive the differential cross section, dσ/dΩ, for electromagnetic elastic scattering of a non-relativistic spinless point particle (electric charge Z_1 , mass m) from a very massive spinless target point particle (electric charge Z , mass M). Show all your steps.

One of the following integrals may be useful:

$$\int dx\;\; e^{a\,x}\sin(bx) = \frac{e^{a\,x}[a\sin(bx) - b\cos(bx)]}{a^2 + b^2}$$

$$\int dx\;\; e^{a\,x}\cos(bx) = \frac{e^{a\,x}[a\cos(bx) + b\sin(bx)]}{a^2 + b^2}$$

$$\int dx\;\; x\;\; e^{a\,x}\sin(bx) = x\frac{e^{a\,x}[a\sin(bx) - b\cos(bx)]}{a^2 + b^2}$$

$$\int dx\;\; x\;\; e^{a\,x}\cos(bx) = x\frac{e^{a\,x}[a\cos(bx) + b\sin(bx)]}{a^2 + b^2}$$

This is the last page of this exam.