

# Energy, entropy, and the village witch

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The topics of usable energy and entropy have great real-world significance, and generate lively discussion by students when discussed in introductory physics courses. This journal has published many resources to lead such a discussion,<sup>1</sup> and other reviews are available elsewhere.<sup>2</sup> A particularly interesting article from this journal by Michael Sobel in 2007 points out that thermodynamics can be used as a storyline to tie together many disparate topics<sup>3</sup>; as Alan Alda<sup>4</sup>, Randy Olson<sup>5</sup>, and others are making increasingly clear, introducing narratives into our physics classrooms is highly effective. In this note these topics of energy and thermodynamics are discussed in a largely qualitative fashion. Some but not all of the discussion is similar to what is found in introductory textbooks, but at the end we will extend these ideas to some situations which are not discussed in textbooks. The level of discussion has been successful in introductory physics courses for non-physics majors, and has also been used, with minor simplifications in the mathematics, in liberal arts “breadth” courses in physics for upper-year undergraduates. There is a storyline involving a village witch.

There is a common perception that having sufficient energy is vital for our future well-being. Thus, when an electrical plant is being designed it is common to see its power output specified as being able to supply enough electricity for some number of thousand homes. However, a brief consideration of the implications of  $E = mc^2$  makes it clear that energy is not the problem: on the earth we currently consume about  $3 \times 10^{20}$  J of energy per year, which is the amount of energy in about 3000 kg of mass. This is a miniscule fraction of the total mass of the earth,  $6 \times 10^{24}$  kg. The problem, then, is not energy, but *usable* energy.

Thermodynamics, and especially its 2<sup>nd</sup> Law, allow us to determine the usable energy of a system. However, as Lidia del Rio and co-authors wrote:<sup>6</sup>

If physical theories were people, thermodynamics would be the village witch. ...  
The other theories find her somewhat odd, somehow different in nature from the rest, yet everyone comes to her for advice, and no-one dares to contradict her.

Witches commonly show themselves in different forms to different people and circumstances. That is true for the village witch of thermodynamics as well. One common statement of the 2<sup>nd</sup> Law is:

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**2<sup>nd</sup> Law of Thermodynamics Form 1.** Heat (thermal energy) “never” spontaneously flows from a colder body to a hotter one.

This form illustrates one aspect of the oddness of the witch: she doesn’t proclaim what will or could happen, but pronounces that something will not happen. We will discuss soon why the word *never* appears in quote marks.

Imagine a pendulum in a box with air molecules, as shown in Figure 1.<sup>7</sup> In the ideal case, the pendulum is an “engine” transforming energy back and forth between gravitational potential energy and kinetic energy. Even in the case of a frictionless pivot of the pendulum, the real pendulum will collide with and transfer energy to the air molecules. So soon the pendulum will stop oscillating, and all of its mechanical energy has been transformed into internal kinetic, i.e. thermal, energy of the molecules.

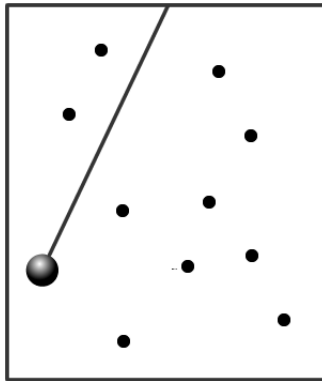


Figure 1. A pendulum in a box with air molecules

As we shall see, this transformation of mechanical energy into thermal energy is irreversible. We will “never” observe all the air molecules colliding with the pendulum and transferring their energy to it, so that the pendulum starts oscillating back and forth again.

Imagine a box containing two black marbles and two white marbles. The system is constrained so that exactly two marbles are on the left and two marbles on the right. We shake the box or otherwise randomize where the marbles are. Figure 2 shows the possible ways that the marbles can arrange themselves. Although we have labeled the marbles, those are just for us: the marbles themselves are identical except for their color.

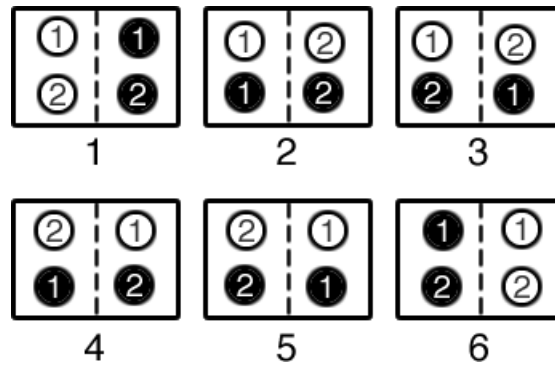


Figure 2. Ways of arranging four marbles in a box.

All six possible arrangements are equally likely. But only one, arrangement 1, has both white marbles on the left and both black marbles on the right, so it will occur  $1/6$  of the time. Similarly only one arrangement, 6, has both black marbles on the left and both white marbles on the right, so this too will occur  $1/6$  of the time. There are 4 arrangements with a one black marble on the left and one on the right, and one white marble on the left and one on the right. So the probability that we will have that 50% of the white marbles are on the left, 50% on the right, and 50% of the black marbles on the left and 50% on the right is  $4/6$ .

We specify the total number of marbles by  $N$ , which is 4 for the example of Figure 2. Then there are  $N/2$  white marbles and  $N/2$  black marbles. We can specify the number of black and white marbles on the left and right by giving, say, the number of black marbles on the left side,  $n$ , where  $0 \leq n \leq N/2$ . Then, the number of states, i.e. arrangements, of the marbles for a given  $n$  and  $N$  is given by:

$$\Omega_N(n) = \left( \frac{(N/2)!}{n! \times (N/2 - n)!} \right)^2 = \binom{N/2}{n}^2 \quad (1)$$

where the exclamation mark indicates a factorial and the second form is the binomial coefficient squared. For  $N = 4$ ,  $\Omega_N(0) = 1$ ,  $\Omega_N(1) = 4$ , and  $\Omega_N(2) = 1$ .

The total number  $C_N$  of combinations of marbles is:

$$C_N = \frac{N!}{(N/2)! \times (N/2)!} = \binom{N}{N/2} \quad (2)$$

For the example of Figure 2  $C_N = 6$ . The probability of a particular value of  $n$  occurring is then:

$$P_N(n) = \frac{\Omega_N(n)}{C_N} \quad (3)$$

For a given value of  $N$ , we can completely specify the number of black and white marbles on the left side and right side by giving, say,  $f$  = the fraction of black marbles on the left side. Then the fraction of black marbles on the right side is  $(1 - f)$ , the fraction of white marbles on the left side is  $(1 - f)$ , and the fraction of white marbles on the right side is  $f$ . Figure 3 shows the probability distributions for  $N = 4$  (blue, squares, thick dashed line),  $N = 20$  (green, triangles, thin dashed line), and  $N = 1000$  (red, dots, continuous line). Note that the probabilities have been re-normalised so they all have the same maximum value.

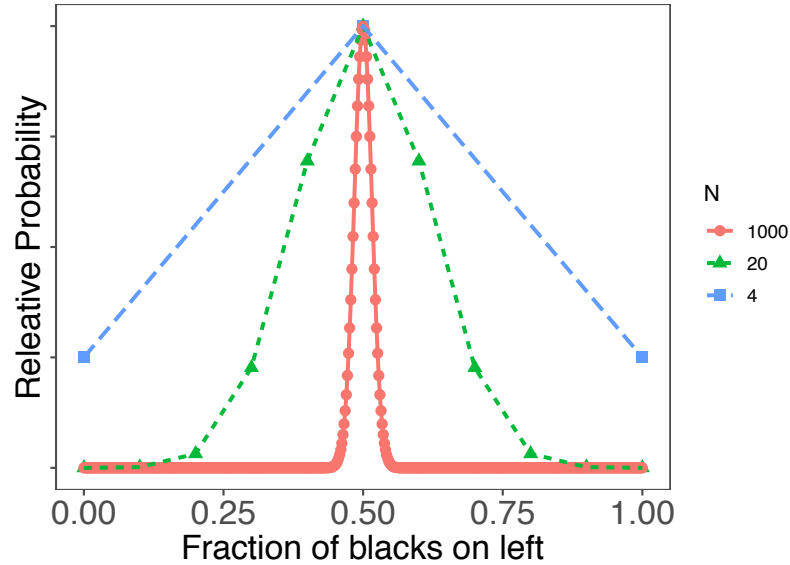


Figure 3. Probability distributions for different values of  $N$ .

For all values of  $N$  the most probable result is one-half of the black marbles on the left, one-half on the right, and with the white marbles also one-half on the left and one-half on the right. However, as  $N$  increases the width of the probability distribution is reduced as approximately  $1/\sqrt{N}$ . This means that as  $N$  become large, the probability of a distribution of marbles without approximately equal numbers in both halves decreases. Table I shows the total probability for the fraction of black marbles on the left being within 0.5% of one-half, i.e.  $0.995 \times 0.5 < f < 1.005 \times 0.5$ . Also listed is the total number of combinations, regardless of the marbles' distribution, from Eqn. 2.<sup>8</sup>

Table I. Probability of the fraction of black marbles on the left being within 0.5% of one-half.

$N$	Probability (%)	Total Combinations $C_N$
1000	15.05	$2.7 \times 10^{299}$
10,000	38.29	$1.6 \times 10^{3008}$
100,000	88.76	$2.5 \times 10^{30,100}$
250,000	98.76	$5.0 \times 10^{75,254}$
500,000	99.96	$1.1 \times 10^{150,512}$
1,000,000	Exactly 100 – $5.7 \times 10^{-7}$	$7.9 \times 10^{301,026}$

Instead of marbles, we imagine a collection of air molecules in a box. Each molecule is moving and colliding with the other molecules and the walls of the container, and this motion is essentially random. The kinetic energy of the molecules, their thermal energy, is proportional to their temperature. We imagine the simple case where the molecules can have just two states: white corresponds to high kinetic energy and high temperature, and black corresponds to low kinetic energy and low temperature. The distribution of marbles that we have been discussing, then, is a good model of these hot and cold molecules. We can now state a second form of the 2<sup>nd</sup> Law:

**2<sup>nd</sup> Law of Thermodynamics Form 2.** The most probable result probably happens.

When we have a collection of hot and cold molecules, the most probable result is that they are both evenly distributed between the left and right sides of the container. This is the case where the two sides have the same temperature. If the box has a volume of  $1 \text{ m}^3$  and contains air molecules at standard temperature and pressure, there are about  $N = 3 \times 10^{22}$  molecules in the box. Since the width of the probability distribution scales as  $1/\sqrt{N}$ , the width of the probability distribution is about  $2 \times 10^{-10}$  times the width of the curve for  $N = 1000$  in Fig. 3. Put another way, for this large value of  $N$ , the probability of the distribution being within 0.5% of exactly one-half, as in Table I, is 100.000% to a huge number of significant figures. We can now see why Form 1 of the 2<sup>nd</sup> Law put the word “never” in quotes: the probability that heat spontaneously flows so the difference in the temperatures in the two halves of the box increases has an extremely small probability, but it is not exactly zero.

For the pendulum of Fig. 1, Mazur in Ref. 7 shows that the most probable state is the one with no energy in the pendulum, and this too scales strongly with the number of air molecules in the box. So once the pendulum has transferred its mechanical energy to thermal energy of the molecules, there is almost no chance that the thermal energy will then transfer back into mechanical energy and the pendulum starts oscillating again.

Feynman<sup>9</sup> gives a definition of disorder as “the number of ways the insides of a system can be arranged so from the outside it looks the same.” We will adopt this definition, although in Ref. 1 Part V Leff has some valid criticisms of this metaphor. For the system of Fig. 2, the four states with equal numbers of black and white marbles on both sides look the same from outside. This definition leads us to a third form of the 2<sup>nd</sup> Law:

**2<sup>nd</sup> Law of Thermodynamics Form 3.** A system “never” spontaneously becomes more ordered.

The 2<sup>nd</sup> Law also tells us how a system is most likely to evolve in time. Thus it tells us the direction of time’s arrow. There are at least two other ways of defining the direction:

1. Expansion of the universe: the universe will be bigger tomorrow than it was yesterday.
2. Consciousness: we remember yesterday but not tomorrow.

It is fun to muse about whether these three ways of determining the direction of time are related.<sup>10</sup>

The thermodynamic village witch is fond of incantations involving the entropy  $S$ . She has a number of ways of defining  $S$ .

For example, we imagine adding a small quantity of heat  $\delta Q$  to a reservoir. If  $\delta Q$  is small enough and the reservoir is large enough that the absolute temperature  $T$  of the reservoir doesn't change, then the change in entropy of the reservoir is:

- $$\delta S = \frac{\delta Q}{T} \tag{4}$$

This is our first definition of entropy: it is in terms of its change.

We imagine heat  $\delta Q$  being removed from a high temperature reservoir at temperature  $T_{\text{hot}}$  and being deposited in a low temperature reservoir at temperature  $T_{\text{cold}}$ . Then the net change in entropy is:

$$\begin{aligned} \delta S_{\text{net}} &= \frac{-\delta Q}{T_{\text{hot}}} + \frac{\delta Q}{T_{\text{cold}}} \\ &= \frac{\delta Q(T_{\text{hot}} - T_{\text{cold}})}{T_{\text{hot}}T_{\text{cold}}} > 0 \end{aligned} \tag{5}$$

If instead the heat  $\delta Q$  is going from the cold reservoir to the hot one, which “never” happens:

$$\begin{aligned} \delta S_{\text{net}} &= \frac{\delta Q}{T_{\text{hot}}} + \frac{-\delta Q}{T_{\text{cold}}} \\ &= \frac{\delta Q(T_{\text{cold}} - T_{\text{hot}})}{T_{\text{hot}}T_{\text{cold}}} < 0 \end{aligned} \tag{6}$$

So the case that “never” happens causes a net decrease in the entropy of the system.

Another equivalent definition of the entropy is from Eqn. 1.

- $$S = k \ln(\Omega_N(n)) \tag{7}$$

where  $k$  is the Boltzmann constant. For the example of Fig. 2, the highest entropy is when  $n = N/4$ .

Finally, because of our definition of disorder a qualitative definition of the entropy is:

- S is a measure of the disorder of a system.

Regardless of the definition of entropy that we use, we can now state a fourth form of the 2<sup>nd</sup> Law:

**2<sup>nd</sup> Law of Thermodynamics Form 4.** The entropy of a system “never” spontaneously decreases.

Most regular physics includes the idea of various physical quantities that are conserved. Here again we see the oddness of the thermodynamic village witch. She is asserting that a physical quantity, the entropy, is not conserved.

The word *spontaneously* appears in Form 1 and Form 4 of the 2<sup>nd</sup> Law, and is crucial. For example, consider a gravel road. When first graded, crushed limestone of a variety of sizes is laid down essentially randomly on the surface. Yet over time the road becomes rutted, with horizontal hills of stones and valleys of the hard subsurface below forming a “washboard”, as shown in Figure 4.<sup>11</sup> The ruts are initially shallow, but get rougher and deeper over time and have a characteristic spacing.

This might seem to be a violation of the 2<sup>nd</sup> Law: the disordered gravel has become more ordered. However, this did not occur spontaneously. The ruts developed from traffic, especially near curves, where cars and truck had to brake to slow down and then accelerate again. So although the gravel became more ordered, this was more than matched by the increase in disorder, mostly thermal energy, of the cars and trucks.



Figure 4. Ruts in a gravel road.

Another example is the formation of sand dunes in the desert. Figure 5, also from Ref. 11, illustrates.



Figure 5 Sand dunes in the desert.

The sand has become ordered into ripples. However the ripples were caused by wind blowing over the desert. The increase in order of the sand was accompanied by a larger increase in the disorder of the wind.

Another example is the evolution of life of earth. About 4 billion years ago there was no life on earth, and today there is about  $5.5 \times 10^{14}$  kg of comparatively highly ordered biomass on the planet. But here too the word *spontaneously* does not apply: the sun provides about  $1.6 \times 10^{17}$  W of power to the earth, and a nearly equal amount is radiated away into space. So energy is streaming through the system of the earth.

A useful model of the earth is a *terrarium*. A quantity of soil, water, plants, and perhaps a couple of snails are placed in a sealed glass container. If the terrarium is placed in a sunny window it can thrive for decades. However, if the terrarium is placed in a dark closet, soon the biomass will have degraded into a foul-smelling high-entropy goo.

This example allows us to ask the thermodynamic village witch about appropriate energy sources for us to use. She mumbles a reply: that to maintain our ecosystem, we must restrict ourselves to energy sources that do not treat the earth as a closed system. Solar power in all its forms qualifies. Examples of solar power include:

- Solar panels to generate electricity.
- Hydroelectric. This uses water flowing downhill to generate electricity. But it is solar power that evaporates the water in the oceans into the atmosphere, where it falls onto the top of the hill as rain. So the energy from water flowing downhill is from the sun.
- Wind power. The sun heats the air near the equator, which causes it to rise. It



then cools near the poles. So the air circulates, flowing from the equator to the poles in the upper atmosphere, and flowing from the poles to the equator nearer the earth's surface. The earth's rotation causes the flow to swirl, but the energy in the wind is from the sun.

- Biomass. We extract energy from biomass in a variety of ways, including burning wood, extracting ethanol from it, and more.

All of these energy sources unclothe the earth, and are sustainable and renewable.

Here are some other energy sources that we use:

- Petroleum, coal, natural gas. These energy sources are ultimately from the sun: biomass from hundreds of thousands of years ago became trapped in the earth as oil, coal, and natural gas deposits. However, these energy sources are not renewable: they are no longer being produced. Thus they all treat the earth as a closed system. The 2nd Law predicts, correctly, that these energy sources will necessarily produce problems with entropy: some common names for this entropy are *pollution*, *greenhouse gases*, and *global warming*.
- Nuclear fission. This treat the earth as a closed system, and the 2<sup>nd</sup> Law guarantees that it increases the entropy of the earth. Among the ways that this entropy manifests are the radioactive tailings produced when uranium is mined, and the large quantity of long half-life radioactive waste produced by the reactors.

Therefore, producing energy from petroleum, coal, natural gas, and nuclear fission are wrong in principle.

Note that the source of the sun's energy is also nuclear. However, the sun is some 150,000,000 km away from the earth. So if we just think about the earth, the energy streams through and we can "beat" the 2<sup>nd</sup> Law. Of course the total entropy of the sun plus the earth plus the rest of outer space is increasing.

There is no doubt that global warming is one of if not the single most urgent problem facing us today. Regardless of whether or not it is caused by humans, it is vital for us to immediately take steps to reduce the phenomenon if we are to survive. This means that eliminating fossil fuels as an energy source must happen now. However, to avert an economic catastrophe they must be immediately replaced with other energy sources. It is possible that increasing our reliance on nuclear fission to produce energy is a necessary alternative in the short term.<sup>12</sup> However, if we increase our reliance on nuclear power, if we listen closely we will hear the thermodynamic village witch cackling.

## References

<sup>1</sup> For example a five part article by H.S. Leff, “Removing the mystery of entropy and thermodynamics,” *The Phys. Teach.* **50**. Part I: pg. 28 (Jan. 2012). Part II: pg. 87 (Feb. 2012). Part III pg. 170 (March 2012). Part IV: pg. 215 (April 2012). Part V: pg. 275 (May 2012).

<sup>2</sup> For example N. Wolchover, “The Quantum Thermodynamics Revolution,” *Quanta Magazine*,

<https://www.quantamagazine.org/the-quantum-thermodynamics-revolution-20170502/>

(Retrieved October 2, 2018).

<sup>3</sup> M. Sobel, “Teaching Thermodynamics and the Nature of Matter,” *The Phys. Teach.* **45**(8), 511 (29 October 2007).

<sup>4</sup> See, for example, the Alan Alda Center for Communicating Science web site at:

<https://www.aldacenter.org/alda-method@>

<sup>5</sup> R. Olson, “Science Communication: Narratively Speaking,” *Science* **32**(6163), 1168 (6 December 2013).

<sup>6</sup> J. Goold, M. Huber, A. Riera, L. del Rio, and P. Skrzypczyk, “The role of quantum information in thermodynamics – a topical review,” *J. Phys. A* **49** (14), 143001 (2016).

<https://doi.org/10.1088/1751-8113/49/14/143001>

<sup>7</sup> This is similar to E. Mazur, *Principles and Practice of Physics*, Vol 1, (Pearson, Toronto, 2015), 504.

<sup>8</sup> Hand-held calculators and most software are not capable of dealing with the huge numbers involved in these calculations. I used *Mathematica*.

<sup>9</sup> R.P. Feynman, R.B. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Volume I, (Addison-Wesley, Toronto, 1963), 46-7.

<sup>10</sup> For example, D. Harrison, “Entropy and the number of sentient beings in the universe,” *Speculations in Science and Technology* **5**, (1982) 43. A brief summary is the final section of D.M. Harrison, “Black Hole Thermodynamics,” (1999):

<https://faraday.physics.utoronto.ca/PVB/Harrison/BlackHoleThermo/BlackHoleThermo.pdf>

<sup>11</sup> Taken by Donald Simanek of Lock Haven University and used by permission.

<sup>12</sup> A passionate argument in favor of this is S. Pinker, *Enlightenment Now*, (Viking, New York, 2018), Chapter 10 “The Environment”.

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