CURVED AIR TRACK

NON CORE

☞You may find the complete guide sheet at
http://faraday.physics.utoronto.ca/IYearLab/cairtrac.pdf

This experiment is an adaptation of The Air Track to simple harmonic motion. This air track is shaped as an arc of a circle of large radius in the vertical plane. Thus a glider on the track behaves like the bob of a simple pendulum of large radius and long period. This enables simple harmonic motion to be "slowed down" for more leisurely observation. Moreover, any object placed on the moving glider is situated in an accelerated frame of reference. The experiment thus allows simple observation of motion of objects in such a frame. After qualitatively observing the motion of the glider on the track, you can measure the radius of curvature of the track by raising an end by a known amount and then observing how much the equilibrium position changes.

Once you know the radius of curvature, determining the period of oscillation gives you enough information to measure the acceleration due to gravity g. You may observe what happens to objects placed on the moving glider.

D. Halliday, R. Resnick, and J. Walker. Fundamentals of Physics, 6th edition. Wiley, 2003.
R.A. Serway. Physics for Scientists and Engineers, 4th edition. Saunders College Publ., 1996.

THE AIR TABLE

NON CORE

☞You may find the complete guide sheet at *http://faraday.physics.utoronto.ca/IYearLab/airtable.pdf*

The air table provides a surface where pucks can move almost without friction. Hence, it can be used to study almost any sort of two-dimensional phenomenon where friction is an unwanted effect, a simple example being the elastic collision of two pucks.

Many of the one dimensional experiments described in The Air Track write-up (see this lab manual) can be adapted to the air table and done **in two dimensions**. Look at that experiment for ideas and then discuss specific plans with your demonstrator.

The equipment consists of an air table, pucks (magnetic or regular in various sizes), masses, pulleys, velcro collars, camera, and VCR plus monitor.

Experiments: The setup will record positions at given times, which one can use to deduce velocities and accelerations. Together with measurements of mass and dimensions, this should be enough to test many principles of mechanics.

References

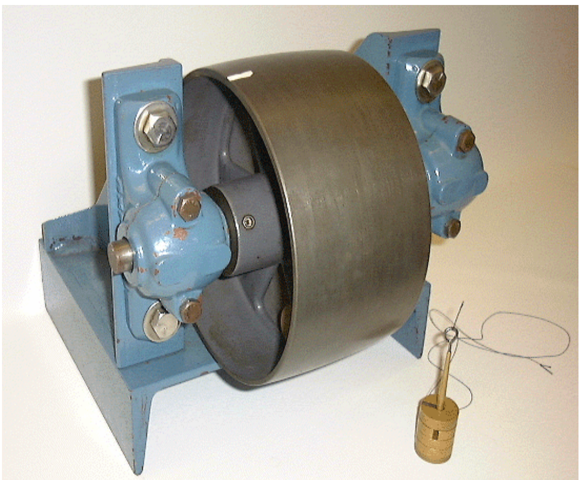
D. Halliday, R. Resnick, and J. Walker. Fundamentals of Physics, 4th edition, vol. 1. Wiley, 1993.
R.A. Serway. Physics for Scientists and Engineers, 4th edition. Saunders College Publ., 1996.

THE FLYWHEEL

REFERENCES

D. Halliday, R. Resnick, and J. Walker. Fundamentals of Physics, 6th edition. Wiley, 2003.

INTRODUCTION



This experiment is an introduction to some basic concepts of rotational dynamics. A fairly realistic analysis of the motion of a flywheel can be made, assuming only that the net frictional torque on a rotating flywheel is constant. In performing this experiment, you will develop understanding of:

- rotational dynamics;
- evaluation of errors in measurements that may be difficult to obtain;
- estimation of a geometrically calculated quantity using simplified models.

THEORY

The basic equations for angular motion can often be obtained simply from those for linear motion by making the following substitutions:

Linear variables	$\Rightarrow$	***Angular variables***
Force, $\vec{F}$	$\Rightarrow$	Torque, $\vec{\tau}$
Mass, m	$\Rightarrow$	Moment of Inertia, I
Velocity, $\vec{v}$	$\Rightarrow$	Angular velocity, $\vec{\omega}$
Momentum, $\vec{p}$	$\Rightarrow$	Angular Momentum, $\vec{L}$
Acceleration, $\vec{a}$	$\Rightarrow$	Angular acceleration, $\vec{\alpha}$

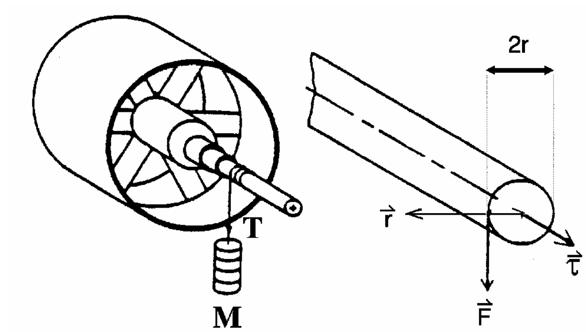
N.B. The analogy needs to be treated with caution. I is not a constant property of the body, as is mass, since its value depends on the axis around which it is measured.

Thus Newton's Law, $\vec{F} = \dfrac{d\,\vec{p}}{dt} = \dfrac{d\,(m\,\vec{v})}{dt} = m\,\vec{a}$, becomes: $\vec{\tau} = \dfrac{d\vec{L}}{dt} = \dfrac{d\,(I\,\vec{\omega})}{dt} = I\,\vec{\alpha}$.

In words, the angular acceleration of a body is directly proportional to the torque applied to it and inversely proportional to the moment of inertia of the body about the relevant axis.

The moment of inertia, **I**, is determined by imagining that the body is divided into a number of infinitesimal elements of mass δm_i each at a distance r_i from the axis of rotation. The moment of inertia **I** about this axis is given by the sum of all the products $(\delta m_i r_i^2)$ calculated for each element, $I = \Sigma_i(\delta m_i r_i^2)$. If the body has a simple geometrical figure, *e.g.* a sphere, cylinder, etc., **I** can be readily calculated and results are tabulated for such bodies in the *CRC Handbook* (available in the lab) or in first year physics texts (see **References**).

The torque $\vec{\tau}$ of a force about an axis is given by the cross-product of the force $\vec{F}$ and the distance from the axis of rotation $\vec{r}$, *i.e.* $\vec{r} \times \vec{F}$. *(Think of opening a door. You are applying a torque whose magnitude equals the product of the force you apply and the distance from your hand to the axis of the door.)*



To summarize: If a body is free to rotate about a fixed axis, then a *torque* $\vec{\tau}$ is required to change the rotational motion of the body and an angular acceleration $\vec{\alpha}$ will result. The angular acceleration is proportional to the net torque (and exists only during the time that the torque acts) and is given by $\vec{\tau} = I\vec{\alpha}$. **I** is a constant of the body, known as its Moment of Inertia about the specified axis of rotation; it depends not only on mass but also on the distribution of mass.

THE EXPERIMENT

In this experiment, a flywheel is mounted so that torques can be applied to it by hanging a mass **M** from the free end of a string, the remainder of which is wrapped around the axle. The torque due to the weight is $\tau = Tr$ where **T** is the tension in the string and **r** the radius of the axle. Because the bearings in the flywheel are not frictionless, there will be a frictional torque exerted at the bearings equal to $\vec{\tau}_f$ which will oppose the motion of the flywheel. Suppose that the string is wrapped around the axle N_1 times and that a mass **M** is suspended from its free end and the system is released at time $t = 0$. The flywheel then is given an angular acceleration, α_1, and the mass **M** accelerates downwards with an acceleration equal to $r\alpha_1$.

If **T** is the tension in the string, then the net torque exerted on the flywheel is:

$$Tr - \tau_f = I\alpha_1$$

The net force on the mass **M** is

$$Mg - T = Ma = Mr\alpha_1$$

Eliminating **T** one finds

$$\alpha_1 = \frac{1}{I} Mgr\left(1 - \frac{r\alpha_1}{g}\right) - \frac{\tau_f}{I} \qquad (1)$$

THE FLYWHEEL

It may be noticed that if the frictional torque, $\boldsymbol{\tau_f}$, is constant, then the angular acceleration of the system, $\boldsymbol{\alpha_1}$, is also constant. With this assumption $\boldsymbol{\alpha_1}$ may be measured experimentally; if $\mathbf{t_1}$ is the time taken for the string to unwind $\mathbf{N_1}$ turns from the axle (*i.e.* the time taken for the mass **M** to drop off the axle), then the flywheel will have rotated through an angle $\mathbf{2\pi N_1}$ radians in the time $\mathbf{t_1}$ and since $\boldsymbol{\theta_1 = \alpha_1 t_1^2/2}$,

$$\boldsymbol{\alpha_1 = 4\pi N_1/t_1^2} \qquad (2)$$

If the flywheel is rotating without an applied torque it will decelerate under the action of the frictional torque alone. Under this condition, Equation 1 becomes

$$\boldsymbol{\alpha_2 = -\,\tau_f/I} \qquad (3)$$

Then if it takes a time $\mathbf{t_2}$ to come to rest, and in this time turns through $\mathbf{N_2}$ revolutions, then the deceleration $\boldsymbol{\alpha_2}$ (assuming constant frictional torque) is given by

$$\boldsymbol{\alpha_2 = -\,4\pi N_2/t_2^2} \qquad (4)$$

If the frictional torque in the bearings is "reasonably" constant, the value of $\boldsymbol{\tau_f}$ obtained by plotting (1) should be consistent with the value obtained from Equation 3. The quantity **I** is by definition a constant and must be the same in Equations 1 and 3. Thus, Equation 3 may be used to independently determine $\boldsymbol{\tau_f/I}$.

POINTS TO CONSIDER

In this experiment you are essentially investigating Equations 1 and 3 using a massive system with little friction.

● Obtain data over as wide a range of values of applied torque as possible (torques up to 0.4 N-m are easily attainable). Also take some data at different values of **r**.

● Are you sure you counted revolutions correctly? What methods of measurement did you use?

● Consider the effect, if any of the thread you use to suspend the weights from the flywheel.

● Equation 1 suggests that fitting of $\boldsymbol{\alpha_1}$ *versus* $\mathbf{Mgr(1-r\alpha_1/g)}$ should be useful. At first sight the error calculation may appear to be a little daunting; however a little thought will convince you that the error in the latter term has only two significant contributions.

> **N.B. If you use *Faraday* to do your fit, we strongly recommended that you use the *massage* and *recalc* facility to calculate any quantities that have long algebraic expressions. If you do the calculation of these quantities at the *fit* stage, a computer glitch often causes printing problems.**

● You may find that the graphing of the data is best done on the computer.

● The derivation of Equations 1 and 3 assumes constant frictional torque, independent of mass **M** and angular speed. Does your data justify this assumption? How would a variation in frictional torque affect your graphs?

● You might try to estimate the value of the moment of inertia **I** for your flywheel by considering a simplified model of its geometry and by finding expressions for **I** (rings, cylinders, rods, etc.). The material of your flywheel is steel and the total mass of its moving parts is 6.5 kg. A rough estimate of the mass of the flywheel contained in the outer cylinder can be made by measuring the approximate volume of the different parts.
(dh -- 1983, jv -- 1988, tk -- 1995,97,98)

Preparatory Questions.

Note: We hope that the following questions will guide you in your preparation for the experiment you are about to perform. They are not meant to be particularly testing, nor do they contain any "tricks". Once you have answered them, you should be in a good position to embark on the experiment.

1. In order to calculate the angular accelerations $\boldsymbol{\alpha_1}$ and $\boldsymbol{\alpha_2}$ using Equations 2 and 4, you need a measurement of the time, **t**, it takes for **N** revolutions. Obviously the errors in **t** and **N** are closely related. Think about a simple approximate way to take account of the error in the values of the angular accelerations that you calculate using these equations. *Hint: You do NOT need to use the formulae for propagation of errors.*

2. The flywheel can be considered to consist of

   i. a hollow outer cylinder,
   ii. a solid inner cylinder, and
   iii. three spokes.

   The contribution to the overall moment of inertia of the flywheel is dominated by the outer cylinder, whose mass is 3 kg. If the length of the outer cylinder is 10 cm and its inner and outer radii are 9.6 cm and 9.9 cm, respectively, calculate an approximate value for the moment of inertia of the flywheel. A*ll values are approximate, and may not correspond very closely to the flywheel you use.*

3. How does the value of $\mathbf{r\alpha_1/g}$ compare with 1, given that the radius of the largest axle is approximately **r** = 3.2 cm and that the largest value of $\boldsymbol{\alpha_1}$ is given in the guide sheet.

4. What implication does your answer to question 3 have for the calculation of the error in the expression $\mathbf{Mgr(1-r\alpha_1/g)}$?

5. Suppose that the frictional torque increased linearly with the load on the axis. What form would you expect the plot of $\boldsymbol{\alpha_1}$ *versus* $\mathbf{Mgr(1-r\alpha_1/g)}$ to take?

THE TORSION PENDULUM

The complete, interactive guide sheet of this experiment can be found at http://faraday.physics.utoronto.ca/IYearLab/Intros/TorsionPend/TorsionPend.html

ABSTRACT

A hollow cylinder hanging from a wire may oscillate in the horizontal plane. We call its ***angular displacement*** from the equilibrium position θ. The wire is twisted at each oscillation and there is a ***restoring torque*** which brings it back to equilibrium. The twisted wire stores potential energy.

The period of a torsion pendulum is given by

$$T = 2\pi\sqrt{\frac{I}{c}}$$

where I is the ***moment of inertia*** of the body about its axis of oscillation and c is the ***torsion constant*** of the suspension wire.

Note the similarity to a spring-mass system undergoing simple harmonic motion

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where m is the mass and k is the spring constant.

In the experiment you will measure the period T for a number of objects with different moments of inertia I. Then a plot of T^2 versus I should be a straight line with slope $4\pi^2 / c$. Thus you can determine the torsion constant ***c*** for the wire.

THE GYROSCOPE

NON CORE

☞You may find the complete guide sheet at
http://faraday.physics.utoronto.ca/IYearLab/gyroscop.pdf

One of the most interesting areas in the science of rotational dynamics is the study of spinning solid objects: tops, hoops, wheels, etc. From the gyrocompass (which indicates *true* North, rather than Magnetic North) to an understanding of how a cyclist turns corners, the applications of this field of study are both practical and fascinating. This experiment is designed to introduce you to some of these interesting and often counter-intuitive properties of rotating bodies.

The apparatus consists of a 2.7 kg cylindrical rotor that is spun at a constant rate by an electric motor. The rotor is mounted in a double gimbal arrangement which allows it to assume any orientation.

First, you will measure the spin angular velocity of the rotor ω (in radians per second). The moment of inertia I can be calculated from the dimensions of the rotor arm, considered as the difference between two solid cylinders. The angular momentum of the rotor around the spin axis, L, can now be found analytically by using the I and ω. Then, you will study two properties of the gyroscope: *precession* and *nutation.*

Diagrams and discussion of the operation of the gyroscope:
The Feynman Lectures on Physics, Chapter 20. Copy available at the Resource Centre.
Operation and Instruction Manual for the MITAC Gyroscope. Copy available at the Resource Centre.

General references:
D. Halliday, R. Resnick, and J. Walker. Fundamentals of Physics, 6th edition. Wiley, 2003.
R.A. Serway and J.W. Jewett. Principles of Physics, 3rd edition. Brooks/Cole, 2002.

OSCILLATIONS OF A SPHERE ON A CONCAVE SURFACE

NON CORE

☞You may find the complete guide sheet at *http://faraday.physics.utoronto.ca/IYearLab/oscofsph.pdf*

On first glance, the motions of a sphere on a concave spherical surface of radius of curvature R might appear to be that of a simple pendulum of length R. However, a quick recognition of the effects of the moment of inertia of the sphere leads to a different conclusion. If a sphere of radius a undergoes small oscillations on a surface of radius R, without slipping, the period is given by

$$T = 2\pi\left[\frac{7}{5}\left(\frac{R - a}{g}\right)\right]^{\frac{1}{2}}$$

You must derive this relation as part of your lab write-up.

This experiment enables a simple precise measurement of the radius R.

The apparatus will allow a measurement of R. This can be best done by using several different spheres on the surface and plotting T^2 *versus* a. The measurement of R can be checked by using the spherometer, which is a purely callipering device.

R.A. Serway and J.W. Jewitt. Principles of Physics, 3rd edition. Brooks/Cole, 2002.

THE WILBERFORCE SPRING

NON CORE

☞You may find the complete guide sheet at
http://faraday.physics.utoronto.ca/IYearLab/WilberforcePendulum.pdf

The Wilberforce spring can undergo two types of harmonic motion:

First, it can oscillate up and down, in which case the period should depend only on the particular spring used and the total mass on the end of the spring.

The other type of harmonic motion possible is a rotational oscillation. The period of the rotational oscillation depends on the particular spring and on the moment of inertia I of the mass (weight plus frame system) on the end of the spring. I can be varied by moving the position of the movable weights.

In this system, these two types of harmonic motion, translational and rotational, are not entirely independent; there is a slight coupling between them. This is because the spring has a slight tendency to coil and uncoil as it is extended or compressed. The Wilberforce spring is thus an example of two weakly coupled resonant systems, other examples being the splitting of energy levels in the ammonia molecule and two simple pendula of similar length with a spring joining the upper parts of their strings. **The Wilberforce spring is a good way to study mechanical resonance in coupled systems.**

D. Halliday, R. Resnick, and J. Walker. Fundamentals of Physics, 6th edition. Wiley, 2003.
R.A. Serway. Physics for Scientists and Engineers, 4th edition. Saunders College Publ., 1996.

BOYLE'S LAW

The complete, interactive guide sheet of this experiment can be found at http://faraday.physics.utoronto.ca/IYearLab/Intros/BoylesLaw/BoylesLaw.html

ABSTRACT

Boyle's Law states that at constant temperature, the pressure p of the gas times its volume V will remain constant:

$$pV = \text{constant.}$$

The above relation is only approximately true. This experiment will allow you to explore this relation of pressure and volume for a fixed quantity of a real gas (dry air) at room temperature and at pressures not very different from normal atmospheric pressure.

You will take indirect measurements of p and V. By using a graphical analysis method, Boyle's Law can be verified and the kinetic energy of the air molecules in volume V can be calculated.

SURFACE TENSION

NON CORE

☞You may find the complete guide sheet at
http://faraday.physics.utoronto.ca/IYearLab/surftens.pdf

The surface of a liquid behaves in some respects as if it were a stretched film or membrane under tension, and the surface tension, T is defined as F/ℓ , where F is the force which the surface exerts on a line of length ℓ in the surface; F is directed at right angles to the line considered. The surface tension of a liquid depends very markedly upon the presence of impurities in the liquid and upon temperature.

This experiment is intended to give you an understanding of:

- The concept of the phenomenon of surface tension.
- Three methods of measuring surface tension, along with a comparison of and critical evaluation of these methods.
- Factors affecting the surface tension.
- The use of certain pieces of apparatus such as a tensiometer, a travelling microscope, and a hydrometer. A hydrometer measures the density of liquids.

D. Halliday, R. Resnick, and J. Walker. Fundamentals of Physics, 6th edition. Wiley, 2003.

VISCOSITY OF WATER BY CAPILLARY FLOW

NON CORE

☞You may find the complete guide sheet at
http://faraday.physics.utoronto.ca/IYearLab/viscosit.pdf

If you have not yet encountered the concept of viscosity in your studies, read the relevant introduction to the subject in one of the references.

The coefficient of viscosity of a fluid can be found from measurements of the volume rate of flow through small tubes. For a simple analysis to be possible, laminar or *streamlined* flow must prevail.

For *streamlined* flow of a liquid through a tube, the volume rate of flow, Q, is given by Poiseuille's formula:

$$Q = \frac{\pi a^4}{8\eta\ell} p \quad \text{or transposing:} \quad p = \frac{8\eta\ell}{\pi a^4} Q$$

where a is the radius of the tube, ℓ its length, and p the pressure difference across its length. Note that this formula reflects the physical equilibrium situation of a force on the fluid due to a pressure difference p being balanced by a viscous force (*i.e.*, one due to a frictional effect), with no other forces acting. J.L.M. Poiseuille, a French physician interested in the circulation of blood, experimentally determined the formula in 1835.

The flow of water, Q, is to be measured at several values of p and for two or three different capillary tubes (available at the Resource Centre). The pressure p depends on the head of water, *i.e.*, the height of the water in the reservoir above the level of the tube.

D. Halliday, R. Resnick, and J. Walker. Fundamentals of Physics, 6th edition. Wiley, 2003.