Racing Cylinders

Introduction

In this Module you will investigate the *moment of inertia* of 2 cylinders, and the effect different moments of inertia have on the the motion of the object.

Duration

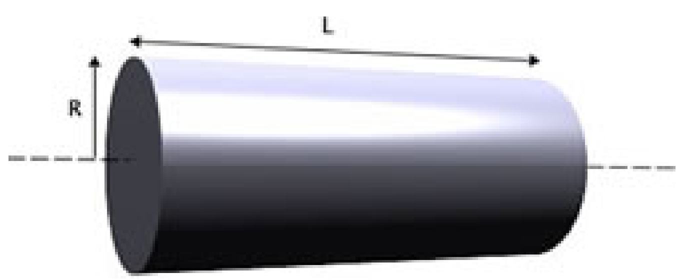
This Module should take about two hours to complete.

Preparation

In addition to looking over this Guide, you should review the topic of rotational motion and the moment of inertia in your textbook.

Background Information

As discussed in your textbook, a solid cylinder of total mass **M**, radius **R**, and length **L** that is rotating about its axis has a moment of inertia **I**:



$$I = \frac{1}{2} M R^2 \qquad (1)$$

Note that **I** does not depend on the length **L**, so Eqn. 1 is also true for a thin disc of negligible thickness.

A hollow cylinder of total mass **M**, inner radius $\mathbf{R_i}$, and outer radius $\mathbf{R_o}$ that is rotating about its axis has a moment of inertia **I**:

$$I = \frac{1}{2} M \left(R_o^2 + R_i^2 \right) \quad (2)$$

The Practical

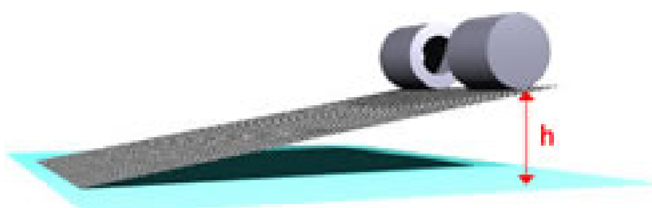
You are given two cylinders, one solid and the other hollow. First, measure the mass and radii of the 2 cylinders. Note that the masses and outer radii are essentially identical.

To measure the radii you will want to use a *vernier caliper*. If you do not know how to use one, a small document on this can be accessed by clicking the yellow button to the right. It is in html format, includes a Java applet, and has a total file size of 57k.

Part 1 - Qualitative

Question 1: Comparing Eqns 1 and 2 above indicates that if you have 2 cylinders, one solid and the other hollow, if they have identical masses and outer radii, the hollow cylinder has a larger moment of inertia. Devise the simplest complete explanation that explains this fact that all members of your Learning Team are comfortable with.

You will be rolling the cylinders down inclined planes.



Question 2: Imagine that the 2 cylinders are rolled side by side down an inclined plane, both starting at the same vertical height and released from rest at the same time. *Predict* which cylinder will reach the bottom of the plane first. Devise an explanation that all members of your Learning Team are comfortable with. You may find thinking about the problem in terms of conservation of energy is simpler than analysing in terms of the torques, forces and accelerations. This approach will lead you to figuring out which cylinder has the greatest speed at the bottom of the plane, which you can then relate to which cylinder reaches the bottom first.

Now you will test your prediction. You have 2 tracks. Raise one end of each the same amount to form two equal inclined planes: the angle each makes with the horizontal should be 10 degrees or so. Release the 2 cylinders from rest at the same vertical heights and observe which one reaches the bottom first. Was your prediction in Question 2 correct? If not, why?

Say you have 2 spheres of identical masses and radii, but one is solid and the other is hollow. Now you cannot *see* any difference between them, but the same experimental test will allow you to determine which is which.

Part 2 - Algebraic

When an object rolls without slipping, if the speed of its centre of mass is **V** and its radius is **R**, then its angular speed ω is given by:

$$\omega = \frac{V}{R} \qquad (3)$$

However, when an object rolls without slipping the point in contact with the surface is always at rest. You may see a Flash animation of this effect by clicking on the blue button to the right.

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Thus, although Eqn. 3 looks like the simple relationship between speed and angular speed for an object in circular motion with a radius **R**, it is not.

Question 3: Convince yourselves that Eqn. 3 is correct.

Question 4: A cylinder with mass **M** and moment of inertia **I** rolls down an inclined plane. If it is released from rest, after its *vertical* distance has changed by an amount **h** derive a relation between the final speed **V** and the mass and moment of inertia. You will want the final answer to give **I** as a function of the other parameters.

Part 3 - Quantitative

Calculate the moments of inertia of the 2 cylinders from your measurements of their masses and sizes.

Now use the Motion Detectors to measure the final speed of the rolling cylinders released from rest after their vertical distance has changed by an amount **h**. Use this data to determine the moments of inertia of the cylinders. Compare your results to the direct calculation.

Why This is For the Birds

If we consider the moment of inertia of a bird's wing, then the lower the value the easier it is for the bird to get the wing to flap. This means that the mass of the wing should be small and should be concentrated close to the axis of rotation. However, there is another requirement: the wing must be strong enough that it does not bend too much or break when the bird is flying, which means the bones of the wing must have sufficiently large masses. Thus there is a trade-off between low moment of inertia and having reasonable structural strength.

The moments of inertia of the wings of 29 bird species and 3 bat species have been measured. It was found that if **I** is the moment of inertia, **M** is the mass and **L** is the length of a wing:

$$I = 0.118\,(M\,L^2)^{(1.040 \pm 0.023)} \qquad (4)$$

Just for interest, you can see the article from the *Journal of Experimental Biology* that did this study by clicking on the button to the right. It is in pdf format, has a size of 143k, and is used by permission.

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For the birds, it was also found that if $\mathcal{P}$ is the power required to flap the wings, it scales as the total body mass **m** according to:

$$\mathcal{P} = 7.75\ m^{\left(0.80\,^{+0.35}_{-0.24}\right)} \tag{5}$$

This is interesting because for organisms from microbes to whales is has been found that the *basal metabolic rate* **bmr** scales as the total body mass as:

$$\mathrm{bmr} \propto m^{0.75} \tag{6}$$

This is consistent with the result of Eqn. 5. Eqn. 6 is called *Kleiber's Law*. You may learn more about this by clicking on the green button to the right. It is in html format and has a file size of 22k including figures.

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Related Experiments

When the *Discovery Practicals* begin, some experiments that also relate to the moment of inertia are:

- Torsion Pendulum
- Flywheel
- Gyroscope
- Wilberforce Spring

Equipment

- 2 Force and Motion Tracks, 2.2 m length, including end stops and rod clamps.
- 2 ring stands.
- 2 Motion Sensors, PASCO CI-6742 or equivalent.
- DataStudio software and 750 Interface unit.
- 2 cylinders. One cylinder is hollow, the other is solid. Both cylinders have identical masses, outer radii and lengths, so they are made of different materials. The lengths should be ~ 10 cm, outer radii ~3 cm.
- Vernier caliper
- Access to a balance.

This module was written by David M. Harrison in April 2005.
The last revision was on $Date$ (y/m/d UTC).