Radioactive Half-Life: Instructor Notes

This document contains notes for the Instructor on the Radioactive Half-Life Module. The contents should not be made available to students.

Preparation

Be sure to look over the Guide to the Module before the Practical. Determine whether your students' textbook discusses radioactivity in terms of the *decay constant* λ or the *lifetime* τ.

Question Answers

Question 1

The probability is 1/6 = 17%

Question 2

Throwing dice is not truly random. If we knew the initial conditions of the throw perfectly, the exact position of where it lands on the tray, the coefficients of restitution of the die and the tray, the viscosity and density of the air, etc. in principle we could calculate which face will be up at the end.

Question 3

I don't think there is necessarily a right answer to this question. Using the theoretical values allows the students to construct a curve of the theoretical values, which will be of the form:

$$N = N_0 e^{-\lambda t}$$

However, using the experimental value allows the students to compare the prediction to the measured value for each individual Trial.

■ Question 4

We expect 5/6 of the cubes to remain after each trail. Thus for Trial 1, we have:

$$\frac{5}{6}\,100 = 83.33$$

After Trail 2 we have:

$$\frac{5}{6}\,83.33 = \frac{5}{6}\left(\frac{5}{6}\,100\right) = \left(\frac{5}{6}\right)^2 100 = 69.44$$

Thus after **n** trials:

$$\left(\frac{5}{6}\right)^n 100$$

cubes remain. So the answer is the value of **n** for which:

$$\left(\frac{5}{6}\right)^n 100 = 50$$

Solving for **n** gives:

$$n = 3.80$$

We would not necessarily expect **n** to b an integer.

■ Question 5

If each trial occurs in 60 seconds, then, the half-life is:

$$n\ \mathrm{trials} * 60\ \mathrm{seconds} / \mathrm{trial} = 228\ \mathrm{seconds}$$

■ Question 6

From Eqn 5 of the Guide:

$$\lambda = \frac{0.693}{T_{1/2}} = 0.00304\ s^{-1}$$

■ Question 7

A bell-shaped or Gaussian shape.

Question 8

$$\sigma = \sqrt{\bar{N}_{\text{estimated}}}$$

Question 9

$$1/3 = 33\%$$

Question 10

After each Trial we expect 2/3 of the dice to remain. Thus:

$$\left(\frac{2}{3}\right)^n 100 = 50$$

$$n = 1.71$$

$$n \text{ trials} * 60 \text{ seconds}/\text{trial} = 103 \text{ seconds}$$

Question 11

$$\lambda = \frac{0.693}{T_{1/2}} = 0.000675 \text{ s}^{-1}$$

Possible Further Topics for Discussion

Here are some ideas for things that you may wish to lead a Learning Team to think about. The last two are probably only suitable for Learning Teams that are keen on mathematical descriptions.

1. What if We Represent a Decay by a Die Coming up 4, 5 or 6?

Then the half-life is the time of one Trial.

2. Does God Play Dice With the Universe?

If he does not, then so-called *identical* nuclei have some *hidden variable* inside them that governs when they will decay.

You might want to discuss the idea of hidden variables with the entire group. Bell's Theorem and its experimental tests have shown that if hidden variables exist, then they are pretty strange. In particular, a hidden variable is in instantaneous contact with every other hidden variable in the universe, and the influences between them propagate at instantaneous speeds.

Einstein has been dead for 50 years, so is not able to defend himself. Nonetheless I think that if he were alive today, Bell's Theorem would pretty well force him to accept Quantum Mechanics.

■ 3. Sampling Experimental Data

The students have been using *DataStudio* software and an Interface unit to collect data for many Modules all year. You may wish to guide a Learning Team to a discussion of how what that hardware/software does is similar to what they have been doing manually in this Module. Here the sampling time Δt is a fake: we *assume* that the samples are collected every 60 s. For the hardware/software the sampling interval is real and set with the software. The Flash animation accessed via the Guide to this Module also has a fixed sampling time: it "measures" the number of remaining atoms every 1/12 s.

For this Module, then, we have worded the Guide rather carefully. For every trail, "when a die has a six facing up, that this corresponds to it having decayed since" the previous trial. Do the students really understand what is going on with sampling intervals?

■ 4. The Relationship Between Decay Probability, Half-Life and Decay Constant

The students will have calculated the half-life for 2 different probabilities of decay in 60 seconds. What is the mathematical relation between the probability, the half-life and the decay constant?

If the probability for decay in a Trial is **P**, then the half-life is:

$$P^n = 0.5$$

So:

$$T_{1/2} = n * 60\,s = \frac{\ln(0.5)}{\ln(P)} * 60\,s$$

The decay constant is:

$$\lambda = \frac{-\ln(0.5)}{T_{1/2}} = \frac{-\ln(P)}{60\,s}$$

■ 5. The Probability Equation and the Exponential Equation for Decays

How do we get from the form:

$$N = N_0\,P^n$$

which is what we have been using to:

$$N = N_0 e^{-\lambda t}$$

Call the time per trial t_n, which here we have been assuming is 60 s. Then:

$$n = \frac{t}{t_n}$$

So:

$$N = N_0 P^{t/t_n}$$

$$\frac{N}{N_0} = P^{t/t_n}$$

Taking the logarithm of both sides gives:

$$\ln\left(\frac{N}{N_0}\right) = \frac{t}{t_n} \ln(P)$$

But in Topic 3 we showed that:

$$\lambda = \frac{-\ln(P)}{t_n}$$

Thus:

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

Author

These notes were written by David M. Harrison in March, 2005.