Mechanics Test: Long Answer

General principles:

Any question with the right answer gets full marks in all cases.
Any mistake correctly used in later parts is only penalised once.
Any correct method gets full marks; my solutions are what I anticipate to be the most common method.
In case of students using other methods, be sure to use a consistent marking scheme and let me know what it is.

■ Part A

The work done on the object is:

$$m_{cyl}\, g h$$

This is all translated into translational kinetic energy:

$$m_{cyl}\, g\, h = \frac{1}{2} m_{cyl}\, v^2$$

Thus:

$$v = \sqrt{2\, g\, h}$$

■ Part B

The work done on the object is:

$$m_{cyl}\, g\, (h - 2\, r_{loop})$$

This is converted to both translational and rotational kinetic energy:

$$m_{cyl}\, g\, (h - 2\, r_{loop}) = \frac{1}{2} m_{cyl}\, v^2 + \frac{1}{2} I_{cyl}\, \omega^2$$

Substituting for I_{cyl} and using that:

$$\omega = \frac{v}{r_{cyl}}$$

we get:

$$m_{cyl}\, g\, (h - 2\, r_{loop}) = \frac{1}{2}\, m_{cyl}\, v^2 + \frac{1}{2}\left(\frac{1}{2}\, m_{cyl}\, {r_{cyl}}^2\right) \frac{v^2}{{r_{cyl}}^2}$$

The mass of the cylinder cancels, so:

$$g\, (h - 2\, r_{loop}) = \frac{3}{4}\, v^2 \qquad (1)$$

For the cylinder to just remain in contact with the track at the top of the loop, the total force on it is just the force due to gravity, and this must be the mass times the centripetal acceleration:

$$m_{cyl}\, g = m_{cyl}\, \frac{v^2}{r_{loop}}$$

The mass cancels so:

$$v^2 = g\, r_{loop}$$

Substituting this into Eqn (1) gives:

$$g\, (h - 2\, r_{loop}) = \frac{3}{4}\, g\, r_{loop}$$

The acceleration due to gravity **g** cancels so, finally:

$$h = \frac{3}{4}\, r_{loop} + 2\, r_{loop}$$

$$h = \frac{11}{4}\, r_{loop}$$

■ Part C

From Part A:

$$v = \sqrt{2\, g\, h} = \sqrt{2 * 9.8\, m/s^2 * 7.3\, m}$$

$$v = 12\, m$$

A wrong answer to Part A with the numbers put into it correctly here: full credit.
No unit in the final answer: no credit.
Do not apply any penalty for too many significant figures in the answer.