Class 15 - November 2/05
Laboratory Topic: Error Analysis

2 Kinds of Statements

1. Exact: \(2 + 3 = 5\)

   \[ \vec{p} = m \vec{v} \]

   \(\text{defn}\)

2. Approximate:

   \[ F_{\text{spring}} = -k \Delta s \]

   \(\text{physical law}\)

   \(g = 9.80 \text{ m/s}^2\)

   \(\text{any quantity about real world}\)
Example: measure period $T$ of pendulum

method: measure time for 5 oscillations $t_5$

$$T = \frac{t_5}{5}$$

$t = 5.97s$ ?

$t = 7.53s$ ✓

$t = 7.38s$

d = 15.2 cm = \frac{1}{2} g t^2$

- Carl Friedrich Gauss (~1800)
Astronomer known for his math.

- discovered that repeated measurements of the same thing produce a distribution of similar results.
- distribution shape is a fit to a histogram called a "Gaussian":

\[ N(x) = A e^{-\frac{(x-x)^2}{2\sigma^2}} \]

an exponential

\[ A = \text{some constant} \]
\[ e = 2.7182... \]
\[ \bar{x} = \text{mean or average} \]
\[ \sigma = \text{standard deviation} \]

"sigma" is these numbers can be estimated from the data

idea: 68% of the times you do a measurement you will find \( x \) between:
\[ \bar{x} - \sigma \quad \text{and} \quad \bar{x} + \sigma. \]

- Many distributions in life have Gaussian shapes.
  - repeated measurements
  - People's heights
  - Random Walk

\[
\text{Repeat measurement } n \text{ times}
\]

If \( n < \infty \), can only estimate \( \bar{x}, \sigma \).
For reasonable values of \( n \)

know \( 0 \) to 1 or maybe 2

significant figures

\[ \text{Error in each individual measurement is } 0 \]

Reading Error

1. Digital Instrument

\[ \pm \frac{1}{2} \text{ of last digit} \]

\[ 0.05 \sim 0.0055 \]

2. Analog Instrument

Your guess
Errors: \[ \text{Read Error.} \]

Choose Largest of the Two as THE ERROR in each individual measurement

**Propagation of errors.**

Let's find the mean of several measurements, all with known error: \( x_1 \pm \Delta x \), \( x_2 \pm \Delta x \)

**Rules of propagating errors:**
- sum rule
- product rule
- multiplying by exact constant
- exponent rule

**Mean:**
\[
\bar{x} = \frac{(x_1 \pm \Delta x) + (x_2 \pm \Delta x) + (x_3 \pm \Delta x) + \cdots}{n} \text{ \( n \) values.}
\]

\[
\bar{x} = \frac{x_{\text{sum}}}{n}
\]

**Error in sum:**
\[
\Delta x_{\text{sum}} = \sqrt{\Delta x^2 + \Delta x^2 + \Delta x^2 + \cdots \text{ \( n \) values}}
\]

\[
= \sqrt{n \Delta x^2}
\]

**Error in mean:**
\[
\Delta \bar{x} = \frac{1}{n} \Delta x = \frac{\sqrt{n}}{n} \Delta x
\]
Significant figures.

- If we calculate a number using other approximate numbers the calculated value may have many digits.

- The error in calculated value determines number of these digits that matter, or signify most likely answer.

i.e.) drop an object from rest: \( d = \frac{1}{2} gt^2 \)

\[
g = 9.80 \text{ m/s}^2 \quad t = 1.2 \quad \rightarrow \quad 2 \text{ sig figs.}
\]

Calculated: \( d = 7.056 \text{ m} \)

\( d = 7.1 \text{ m} \)

This rule comes from error analysis.