Error Analysis

“To err is human; to describe the error properly is sublime.”
- Cliff Swartz (1999)

Today

☐ A discussion about a laboratory topic: error analysis
☐ Your learning of this:
   1. The assignment
   2. Using error analysis in an experiment
   3. This talk
   4. A test (administered via computer)

Presenters: a “Tag Team”

Coming Next Week…

- We will begin the Waves Quarter on Oscillations, Sound and Light.
- For Monday, please read Sections 14.1 through 14.3 of Knight.
- There is a Pre-Class Quiz (Waves #1) on Chapter 14 due Monday morning on www.masteringphysics.com.
- Don’t forget your voting cards!

The t₅ data

| 7.53 s  |
| 7.38 s  |
| 7.47 s  |
| 7.43 s  |

Histogram: 4 Measurements

- 7.53 s
- 7.38 s
- 7.47 s
- 7.43 s
The Gaussian

\[ N(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x - \mu)^2}{2 \sigma^2}} \]

68% of data between the dotted lines on the graph.

Heights of some People (London, 1886)
Random Walk

Where does an object end up, if it takes N steps randomly left or right? The final distribution is described by a Gaussian function!

The $t_5$ data

Numerically:

<table>
<thead>
<tr>
<th>Time</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.53 s</td>
<td>± 0.06 s</td>
</tr>
<tr>
<td>7.38 s</td>
<td>± 0.06 s</td>
</tr>
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<td>7.47 s</td>
<td>± 0.06 s</td>
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<td>7.43 s</td>
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</tbody>
</table>

$\bar{t}_{5,est} = 7.45250 \pm 0.06 \text{ s}$

$\sigma_{est} = 0.06 \text{ s}$

Propagation of Errors

\[
\begin{align*}
\Delta z &= \sqrt{\Delta x^2 + \Delta y^2} \\
\Delta z &= \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right) z \\
\Delta z &= A \Delta x \\
\Delta z &= |n x^{n-1} \Delta x|
\end{align*}
\]

Repeated Measurements

- Repeated n times
- Each individual measurement has an error of precision $\Delta x$

\[
\Delta \bar{x}_{est} = \frac{\Delta x}{\sqrt{n}}
\]

The $t_5$ Data Again

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$\bar{t}_{5,est} = 7.45 \pm 0.03 \text{ s}$

$T = 1.490 \pm 0.006 \text{ s}$

Significant Figures

- Discussed in Section 1.9 of Knight Ch.1
- Rules for significant figures follow from error propagation
  - Assume error in a quoted value is half the value of the last digit.
  - Errors should be quoted to 1 or 2 significant figures
  - Error determines significant figures of the value.
- Example: If a calculated result is $(7.056 \pm 0.705) \text{ m}$, it is better to report $(7.1 \pm 0.7) \text{ m}$. 