Error Analysis

"To err is human; to describe the error properly is sublime."
- Cliff Swartz (1999)

Today

- A discussion about a laboratory topic: error analysis
- Your learning of this:
  1. The assignment
  2. Using error analysis in an experiment
  3. This talk
  4. A test (administered via computer)

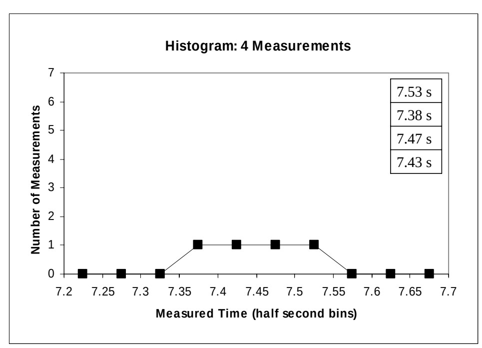
Presenters: a "Tag Team"

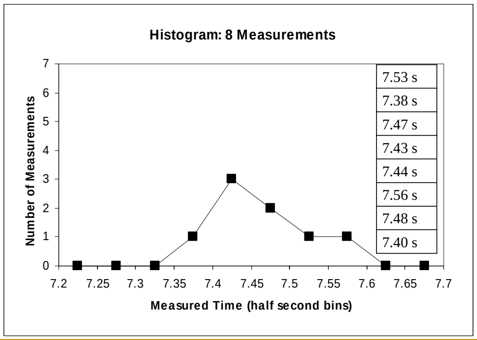
Coming Next Week…

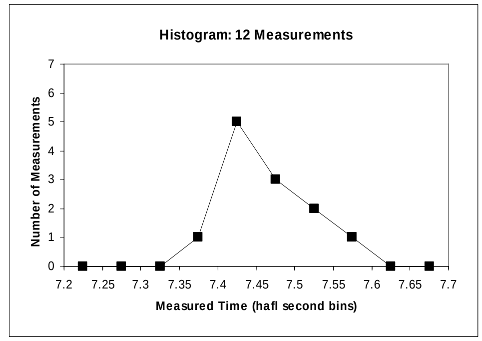
- We will begin the ***Waves Quarter*** on Oscillations, Sound and Light.
- For Monday, please read Sections 14.1 through 14.3 of Knight.
- There is a Pre-Class Quiz (Waves #1) on Chapter 14 due Monday morning on www.masteringphysics.com.
- Don't forget your voting cards!

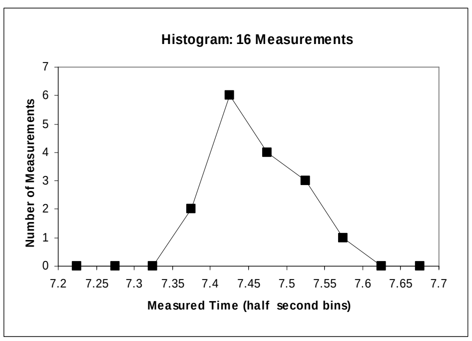
The t_5 data

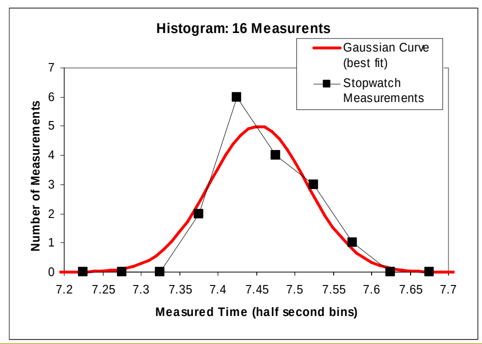
7.53 s
7.38 s
7.47 s
7.43 s





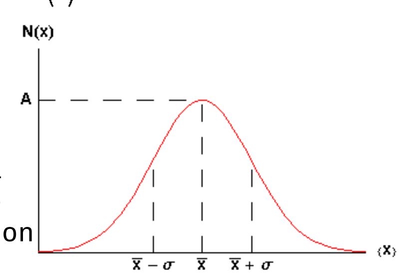






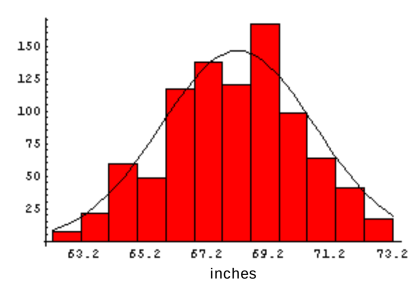
The Gaussian

$$N(x) = A\, e^{\frac{-(x-\bar{x})^2}{2\sigma^2}}$$

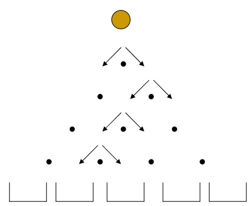


68% of data between the dotted lines on the graph.

Heights of some People (London, 1886)



Random Walk



Where does an object end up, if it takes N steps randomly left or right?

The final distribution is described by a Gaussian function!

The t_5 data

7.53 s ± 0.06 s
7.38 s ± 0.06 s
7.47 s ± 0.06 s
7.43 s ± 0.06 s

Numerically:

$$\bar{t}_{5,est} = 7.45250\ s$$

$$\sigma_{est} = 0.0634429\ s$$

$$\sigma_{est} = 0.06\ s$$

Propagation of Errors

$$z = x + y \qquad z = x - y \qquad \Delta z = \sqrt{\Delta x^2 + \Delta y^2}$$

$$z = x * y \qquad z = x / y \qquad \frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

$$z = A\ x \qquad \Delta z = A\ \Delta x$$

$$z = x^n \qquad \Delta z = |\ n\, x^{n-1}\, \Delta x\ |$$

Repeated Measurements

- Repeated **n** times
- Each individual measurement has an error of precision Δ**x**

$$\Delta \bar{x}_{est} = \frac{\Delta x}{\sqrt{n}}$$

The t_5 Data Again

7.53 ± 0.06 s
7.38 ± 0.06 s
7.47 ± 0.06 s
7.43 ± 0.06 s

Numerically:

$$\bar{t}_{5,est} = 7.45250\ s$$

$$\sigma_{est} = 0.06\ s$$

$$\bar{t}_{5,est} = 7.45 \pm 0.03\ s$$

$$T = 1.490 \pm 0.006\ s$$

Significant Figures

- Discussed in Section 1.9 of Knight Ch.1
- Rules for significant figures follow from error propagation
  - Assume error in a quoted value is half the value of the last digit.
  - Errors should be quoted to 1 or 2 significant figures
  - Error determines significant figures of the value.
- Example: If a calculated result is (7.056 ± 0.705) m, it is better to report (7.1 ± 0.7) m.