Error Analysis

“To err is human; to describe the error properly is sublime.”
- Cliff Swartz (1999)

Today

• A discussion about a laboratory topic: error analysis
• Your learning of this:
  1. The assignment
  2. Using error analysis in an experiment
  3. This talk
  4. A test (administered via computer)

Presenters: a “Tag Team”

Coming Next Week…

- We will begin the Waves Quarter on Oscillations, Sound and Light.
- For Monday, please read Sections 14.1 through 14.3 of Knight.
- There is a Pre-Class Quiz (Waves #1) on Chapter 14 due Monday morning on www.masteringphysics.com.

Two Kinds of Statements

1. Exact
   • \(2 + 3 = 5\) (math)
   • \(K = \frac{1}{2} mv^2\) (definition)
2. Approximate
   • \(F_{\text{spring}} = -kx\) (any physical law)
   • \(g = 9.80 \text{ m/s}^2\) (all numerical measures of the universe)

Today: approximate statements

The \(t_5\) data

<table>
<thead>
<tr>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.53 s</td>
</tr>
<tr>
<td>7.38 s</td>
</tr>
<tr>
<td>7.47 s</td>
</tr>
<tr>
<td>7.43 s</td>
</tr>
</tbody>
</table>
The Gaussian

\[ N(x) = A e^{-\frac{(x - \bar{x})^2}{2\sigma^2}} \]

68% of data between the dotted lines on the graph.
Heights of some People
(London, 1886)

Random Walk
Where does an object end up, if it takes
N steps randomly left or right?
The final distribution is described by a
Gaussian function!

The Gaussian
\[ N(x) = A e^{-\frac{(x-\bar{x})^2}{2\sigma^2}} \]
68% of data
between the
dotted lines on
the graph.

The \( t_s \) data

<table>
<thead>
<tr>
<th>Value</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.53 s</td>
<td>± 0.06 s</td>
</tr>
<tr>
<td>7.38 s</td>
<td>± 0.06 s</td>
</tr>
<tr>
<td>7.47 s</td>
<td>± 0.06 s</td>
</tr>
<tr>
<td>7.43 s</td>
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</tr>
</tbody>
</table>

Numerically:
\[ \bar{t_{s,\text{est}}} = 7.45250 \text{ s} \]
\[ \sigma_{\text{est}} = 0.0634429 \text{ s} \]
\[ \sigma_{\text{est}} = 0.06 \text{ s} \]

A Digital Instrument

Propagation of Errors
\[ z = x + y \]
\[ z = x - y \]
\[ z = x \cdot y \]
\[ z = x / y \]
\[ z = A \cdot x \]
\[ z = x^n \]
\[ \Delta z = \sqrt{\Delta x^2 + \Delta y^2} \]
\[ \frac{\Delta z}{z} = \sqrt{\left( \frac{\Delta x}{x} \right)^2 + \left( \frac{\Delta y}{y} \right)^2} \]
\[ \Delta z = A \Delta x \]
\[ \Delta z = | n \cdot x^{n-1} \Delta x | \]
Repeated Measurements

- Repeated $n$ times
- Each individual measurement has an error of precision $\Delta x$

$$\Delta x_{est} = \frac{\Delta x}{\sqrt{n}}$$

Significant Figures

- Discussed in Section 1.9 of Knight Ch.1
- Rules for significant figures follow from error propagation
  - Assume error in a quoted value is half the value of the last digit.
  - Errors should be quoted to 1 or 2 significant figures
  - Error should be in final displayed digit in number.
- Example: If a calculated result is: $(7.056 \pm 0.705) \text{ m}$, it is better to report: $(7.1 \pm 0.7) \text{ m.}$