Special Topic: Error Analysis
November 1, 2006

2 kinds of statements

(1) Exact

$$2 + 3 = 5$$

$$K = \frac{1}{2} m v^2$$

(2) Approximate

$$F_{spring} = -kx$$ any physical law

$$g = 9.80 \, m/s^2$$ any physical quantity

Example: Determine period T of pendulum

Method: Measure time for 5 oscillations: t_5

$T = \frac{t_5}{5}$

Measurements: ~~5.97~~ s.. ← started at "one"

7.53 s. } started at "zero"
7.38 s.
⋮

Carl Friedrich Gauss (~1800)
Discovered that repeated measurements produce a distribution of answers, whose shape is a "Gaussian" or "normal distribution". ("Bell curve")

Gaussian function is:

$$N(x) = A\, e^{\left[-\frac{(x-\bar{x})^2}{2\sigma^2}\right]}$$

"exponential function"

where $N(x)$ = # of measurements near x (within some bin size)

A = arbitrary constant

e = 2.7182...

$\bar{x}$ = mean or average of x

"sigma" → σ = standard deviation.

Many distributions in life have Gaussian shapes.

ie : - test scores
- heights of people
- random walk
- repeated measurements of the same thing.

σ is the error in each individual measurement.

estimate standard deviation

numerically:

$$\sigma_{est} = 0.0634429\ s$$

$$\overline{t}_{5,est} = 7.45250\ s$$

$$\underline{\sigma_{est} = 0.06\ s}$$

Reading Error

① Digital Instrument

error ± ½ last digit

Not value of

7.56 s error 0.005 s

7.560 ± 0.005 s

Largest of σ or Reading Error

② Analog

It is your eyes.
How well can you read the result?

Error Propagation.

Assuming you have measured x with error Δx. (x ± Δx).
also you have measured y with error Δy (y ± Δy)

You use your two measurements to compute z. What is the error in z?

Rules: Sum Rule

$$z = x + y \quad \text{or} \quad z = x - y$$

error in z: $\Delta z = \sqrt{\Delta x^2 + \Delta y^2}$

Product Rule:

$$z = xy \quad \text{or} \quad z = x/y$$

$$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

Multiply by an exact constant rule:

$$z = Ax \quad , \quad \Delta z = A\Delta x$$

Error in the mean.

if $\bar{x} = \dfrac{x_1 + x_2 + \cdots x_N}{N}$

And errors in individual measurements are Δx,

error in $\bar{x}$ is $\boxed{\Delta \bar{x} = \dfrac{\Delta x}{\sqrt{N}}}$

→ repeated measurements

improve your estimate of the "truth".

"Sig. Figs" A number you report should not have more digits than you actually know within error.

Example: 7.056 ± 0.705

should be reported as:

7.1 ± 0.7