Using Conservation of Energy

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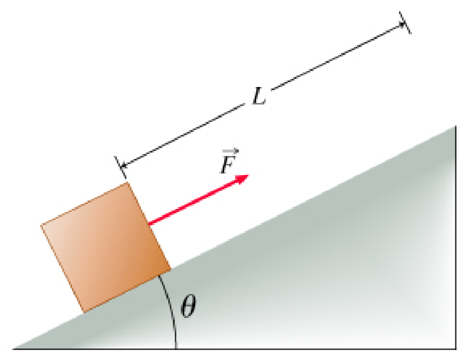
I am still getting a number of questions with a related theme. Many arise from a *MasteringPhysics* question from Chapter 10 and 11. Here I attempt to sort out some of the confusion.

The Question

The question from *MasteringPhysics* is based on the following:

> A block of weight w sits on a frictionless inclined plane, which makes an angle θ with the horizontal, as shown. A force of magnitude F, applied parallel to the incline, pulls the block up the plane *at constant speed*. The block moves a distance L up the incline.



We will use this situation to "derive" that the total force acting on the block along the direction of the incline is zero.

Work-Kinetic Energy Theorem

From Newton's 2nd Law, we proved that the net work W_{net} done on a system is always equal to its change in kinetic energy ΔK. This is Equation 11.10 of the text:

$$\Delta K = W_{net}$$

We apply this to the question by realizing that the net work has two terms:

1. The work done by the external force: $W_F = F\,L$.
2. The work done by gravity: $W_g = -\,w\,L\sin(\theta)$.

Note that the work done by gravity depends on the component of the gravitational force along the incline, $w\sin(\theta)$, and is negative.

Since for this situation the change in kinetic energy is zero:

$$W_{net} = 0$$

Therefore:

$$F\,L = w\,L\sin(\theta)$$

The length L cancels so:

$$F = w\sin(\theta) \qquad (1)$$

Gravitational Potential Energy

When the forces acting on a system are *conservative*, we can describe the work done by them in terms of the *potential energy*, their potential to do work. Of course, as the work is actually done by a conservative force, its potential to do work becomes less. For a gravitational force, then:

$$W_g = -\Delta U_g$$

If all the forces acting on a system are conservative, then we can describe them in terms of their potential energies, and say that the *mechanical energy*, E_{mech}, is conserved.

$$E_{mech} = K + U_g + U_s + \ldots = \text{constant}$$

Then conservation of energy states that for the question we are considering, the work done by the external force is equal to the change in mechanical energy of the block:

$$W_F = \Delta E_{mech} \qquad (2)$$

Since the kinetic energy is constant, the change in mechanical energy is just the change in the gravitational potential energy:

$$\Delta E_{mech} = \Delta U_g = w\,\Delta y$$

Here Δy is the change in the vertical position of the block, and w is its weight. But Δy is just $L\sin(\theta)$, so Equation (2) becomes:

$$F\,L = w\,L\sin(\theta)$$

The length L cancels so:

$$F = w\sin(\theta) \qquad (3)$$

This is exactly Equation (1). So these two ways of applying energy ideas to this question give identical results: the total force acting on the block along the direction of the incline is zero.

A Common "Misteake"

I have seen many students attempt to write something like:

$$W_{\text{net}} = \Delta E_{\text{mech}} \qquad (4)$$

What is wrong with this? The net work acting on the block includes the work done by gravity. But the work done by gravity is also described in terms of its change in gravitational potential energy, which is part of the change in mechanical energy.

If you glance back at the two approaches to the problem that we went through above, you will see that:

- In the **Work-Kinetic Energy Theorem** section, the force due to gravity is described in terms of the work it does on the block, the W_{net} term.
- In the **Gravitational Potential Energy** section, we do not explicitly consider the work done by gravity on the block. Instead we account for that work by describing the change in gravitational potential energy.

Thus, Equation (4) is wrong because it "double counts" the effect of the gravitational force: once on the left hand side and again on the right hand side.

Let's use Equation (4) to get a wrong result. I re-write that equation using the values for the net force and the change in mechanical energy from before:

$$F\,L - w\,L\sin(\theta) = w\,L\sin(\theta)$$

Then:

$$F = 2\,w\sin(\theta)$$

This is clearly wrong.