Solutions

# PHY 238Y - LIFE SCIENCES II

**MID-TERM TEST #1**
**November 12, 2001**
**Time: 50 minutes**

**NAME:**______________________________ **STUDENT NUMBER**____________

**Calculators may be used. All sub-questions have approximately equal mark value, but may not be of equal difficulty.**

**Formulae:**

Ideal Gas Law: $PV = nRT$

Average Kinetic Energy: $K_{ave}=(1/2)m(v^2)_{ave}=(1/2)m(v_{rms})^2=(3/2)k_BT$

Osmotic Pressure: $\pi = cRT$, $c=n/V$

Continuity equation: $Q=Av= const$

Bernoulli's Equation: $P+(1/2)\rho v^2+ \rho gh = const$

Reynolds number: $N_R=\dfrac{2\rho\bar{v}R}{\eta}$

$N_R<2000$ – laminar flow

$N_R>3000$ – turbulent flow

Pressure: $P=F/A$

Hydrostatic pressure: $P=\rho gh$

Poiseuille's law: $Q = \dfrac{\Delta P\pi R^4}{8\eta l}$

Laplace's law: $\Delta P = \dfrac{2\gamma}{r}$

**Constants:**

Universal Gas Constant $R= 8.31$ J mole$^{-1}$ K$^{-1}$

Absolute Temperature: $T(K) =273.15+ T(°C)$

Atmospheric pressure 1atm $=1.013 \times 10^5$ Pa

Viscosity of water $= 1.005 \times 10^{-3}$ Pa s

Density of air $= 1.19$ kg m$^{-3}$

Density of blood $= 1059.6$ kg m$^{-3}$

Density of water $= 1000$ kg m$^{-3}$

Surface tension of water $= 7.28 \times 10^{-2}$ N m$^{-1}$.

A mole of a substance is an amount whose mass in grams is numerically equal to the molecular mass in atomic mass units.

**Question #1**

(i) Assume that the pressure in a room remains constant at $1.01\times10^5$ Pa and the air is composed only of nitrogen ( M=28.01 g mol$^{-1}$). The volume of the room is 60.0 m$^3$. When the temperature increases from 289 to 302 K, what mass of air (in kg) escapes from the room?

$PV = nRT$ T, V are constant

$$\Delta n = n_1 - n_2 = \frac{PV}{R}\left(\frac{1}{T_1}-\frac{1}{T_2}\right) = \frac{1.013\times10^5\,Pa\times 60\,m^3}{8.31\,J/(mol\cdot K)}\left(\frac{1}{289}-\frac{1}{302}\right)\frac{1}{K} =$$

$\Delta n = 101.5$ mole - escaped

Mass: $m = \Delta n \dfrac{M}{1000} \approx 3\,kg$

Answer: 3 kg kg

(ii) Initially, the translational rms speed of a molecule of an ideal gas is 463 m s$^{-1}$. The pressure and volume of this gas are kept constant, while the number of molecules is doubled. What is the final translational rms speed of the molecules?

1) $\frac{mV_{rms}^2}{2} = \frac{3}{2}k_BT$, $V_{rms} = \sqrt{\frac{3k_BT}{m}}$ m – mass of one molecule!

2) $PV = nRT$, $PV = const \Rightarrow n_1RT_1 = n_2RT_2 \Rightarrow n_1RT_1 = 2n_1RT_2 \Rightarrow$

$T_2 = \frac{T_1}{2}$ – temperature changes, $\Rightarrow$ average kinetic energy changes

$$\frac{V_{rms2}}{V_{rms1}} = \sqrt{\frac{T_2}{T_1}} = \frac{1}{\sqrt{2}};$$

$V_{rms_2} = \frac{V_{rms_1}}{\sqrt{2}} = \frac{463\,m/s}{\sqrt{2}} = 327\,m/s$

Answer: 327 m s$^{-1}$

(iii) What concentration difference of impermeable solutes across a cell membrane would result in an osmotic pressure of 5 atm at T=37°C?

$\pi = cRT$

$5\,atm = 5\times 1.013\times10^5\,Pa$

$T = 273+37 = 310\,K$

$$C = \frac{\pi}{RT} = \frac{5\times1.013\times10^5\,Pa}{8.31\,\frac{J}{mol\,K}\times 310\,K} =$$

$= 198\ \frac{moles}{m^3}$

Answer: 198 moles m$^{-3}$

*Notes: for 1(ii) common mistake is to assume $m_2 = 2m_1$, m is the mass of one molecule, it does not change! T and K ave change when n is changed



for 1(iii) $\pi$ must be expressed in Pa in order to obtain the result expressed in moles/m$^3$

**Question #2**

(i) The construction of a flat rectangular roof (4.0 m x 5.5 m) allows it to withstand a maximum net outward force of 21 000 N. The density of the air is 1.19 kg $m^{-3}$. At what wind speed will this roof blow outward?

The force is due to the difference of pressures inside and outside the building: $A = 4 \times 5.5 m^2 = 22 m^2$

$\Delta P = \frac{F}{A}$ ;

Pressure difference due to windy conditions can be found from Bernoulli's equation:

$P_{inside} = P_{outside} + \frac{\rho v^2}{2} \Rightarrow \Delta P = \frac{\rho v^2}{2} \leq F/A$

$v \leq \sqrt{\frac{2F}{\rho A}} = \frac{2 \times 21000 N}{1.19 \frac{kg}{m^3} \times 22 m^2} = 40 m/s$ - upper limit for wind speed

Answer: 40 m $s^{-1}$

(ii) Suppose the blood flow through the aorta with a speed of 0.35 m $s^{-1}$. The cross-sectional area of the aorta is 2.0x$10^{-4}$ $m^2$.

(a) Find the volume flow rate of blood.

$Q = \bar{V}_a A_a = 0.35 m/s \times 2 \times 10^{-4} m^2 = 7 \times 10^{-5} m/s$

Answer: $7 \times 10^{-5}$ $m^3$ $s^{-1}$

(b) The aorta branches into tens of thousands of capillaries whose total cross-sectional area is about 0.28 $m^2$. What is the average blood speed through the capillaries?

Continuity equation:

$\bar{V}_a A_a = \bar{V}_c A_c$

$\bar{V}_c = \frac{\bar{V}_a A_a}{A_c} = \frac{Q}{A_c} = \frac{7 \times 10^{-5} m/s}{0.28 m^2} = 2.5 \times 10^{-4}$

Answer: $2.5 \times 10^{-4}$ m $s^{-1}$

Common mistake: to assume that the exact number of capillaries is known.

We can use the total cross-sectional area of capillaries instead.

**Question #3**

(i) A cylindrical air duct in an air conditioning system has a length of 5.5 m and a radius of $7.2 \times 10^{-2}$ m. A fan forces air (viscosity of $1.8 \times 10^{-5}$ Pa s) through the duct, such that the air in a room (volume = 280 m$^3$) is replenished every ten minutes.

Determine, if **possible**, the difference in pressure between the ends of the air duct.

$Q = \frac{V}{\Delta t} = \frac{280\ m^3}{60s \times 10} = 0.48\ \frac{m^3}{s}$ - flow rate (volume)

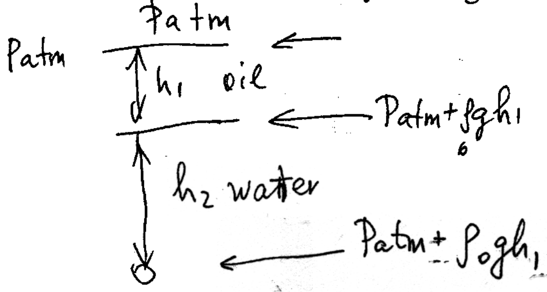
$\bar{v} = \frac{Q}{\pi r^2} = \frac{0.48\ m^3/s}{3.14 \times 7.2^2 \times 10^{-4}\ m^2} = 29.5\ m/s$ - average speed

Reynolds number: $N_R = \frac{2\rho \bar{v} r}{\eta} = 2.8 \times 10^5 \gg 3000$ - turbulent flow, $\Rightarrow$

Poiseuille's law cannot be applied (since it is for laminar flow only)

$\Delta P$ cannot be determined

(iii) Calculate the gauge pressure inside a spherical bubble of radius $8 \times 10^{-5}$ m located in a water tank 15 m below the surface of water. On top of the water there is a 5 m layer of oil of density 500 kg m$^{-3}$.




$P_{atm} + \rho_o g h_1 + \rho_w g h_2 = P_{outside}$

$P_{inside} - P_{outside} = \frac{2\gamma}{r}$ Answer: $1.73 \times 10^5$ Pa

(since the bubble is in the water, only one interface is present)

$P_{gauge} = P_{inside} - P_{atm} = \rho_o g \underset{5m}{h_1} + \rho_w g \underset{15m}{h_2} + \frac{2\gamma}{r} = 1.735 \times 10^5$ Pa