Lecture 1

History, Tools and a Roadmap

James Clerk Maxwell - 1831-1879

- Born in Edinburgh, Scotland
  - 13 November 1831
  - 14 India Street
- Died 5 November 1879
- Declared redundant from U of Aberdeen in 1860
- 1st Cavendish Professor of Physics at Cambridge 1871
- Treatise on Electricity and Magnetism published in 1873
- http://www-groups.dcs.st-and.ac.uk/history/Mathematicians/Maxwell.html

James Clerk Maxwell - 1831-1879

Contributions to Science

- Maxwell's Equations!!
- Predicted Electromagnetic Waves
- Colour in photography
- Kinetic Theory
- Planimeter (mechanical integration machine)
- Repeated experiments of Cavendish
- Advocate of the telephone

Where to M.Es Come From?

- Gauss Law for electric fields
- Integral of the normal component of the electric field is proportional to the total charge enclosed:
- Int[**E**.d**S**] = $\Sigma q / \epsilon_0$ = Int[ρ dv] / ϵ_0
- Use Gauss's theorem to relate surface integral to volume integral:
- Div[**E**] = ρ / ϵ_0
- BUT must include the polarisation charges (if any)
- Define the polarisation field as **P**

Electrostatics

- Including dielectric media
- Div[ϵ_0 **E**] = - Div[**P**] + ρ
- In free space (no material) **P** = 0
- $\epsilon_0 = 10^{-9}/(36\pi)$ in SI units
- Do we often have material (**P** # 0) ? - Yes
- Do we often have free charges (ρ # 0)? - No

Static Magnetic Fields

- Very similar to electrostatic fields
- Div[**H**] = 0
- There are no free monopoles (I think!)
- BUT must include the Magnetisation **M**
- Div[**H**] = -Div[**M**]
- Do we often have magnetic materials (**M** # 0)?
  - All materials polarise to some extent
  - Only some magnetise significantly

Changing Magnetic Fields - Faraday

- Relates a changing magnetic field to a potential
- V = - d/dt (N) (Faraday)
- N is measured in Webers
- but...
- V = Int[**E** . d**s**] (electrostatics)
- and...
- N/μ_0 = Int[(**H** + **M**) . d**S**] (electromagnetics)
- so...
- Int[**E** . d**s**] = $-\mu_0\, \partial/\partial t$ Int[(**H** + **M**) . d**S**]
- = Int[Curl[**E**] . d**S**]
- Curl[**E**/ μ_0] = - $\partial/\partial t$ (**H**) - $\partial/\partial t$ (**M**)
- μ_0 relates the electric and magnetic unit systems
- $\mu_0 = 4\pi \cdot 10^{-7}$ in SI units

Continuity

- Since current is charge in motion
- Div[**J**] = - ∂/∂t (ρ)
- Charge is conserved - confirmed by experiment!!

Ampere's Law

- The integral of the normal component of **H** round any closed boundary is proportional to the integral of the normal component of current on any surface with that boundary
- Cint[**H** . d**s**] = Int[**J** . d**S**]
- by Stoke's theorem
- Curl[**H**] = **J** (Ampere's Law)
- BUT Div[Curl[]] = 0
- SO Div[**J**] = 0 which isn't true!!
- TROUBLE!! - Ampere was wrong!

Maxwell's Contribution

- Div[**J**] = - ∂/∂t (ρ)
- Div[**J**] = - ∂/∂t (Div[ϵ_0 **E**] + Div[**P**])
- Div[**J** + ∂/∂t (ϵ_0 **E**) + ∂/∂t (**P**)] = 0
- Above satisfies continuity
- SO CHANGE AMPERE's LAW!!
- instead of
- Curl[**H**] = **J**
- write...
- Curl[**H**] = ∂/∂t (ϵ_0 **E**) + ∂/∂t (**P**) + **J**
- then Div[Curl[**H**]] = 0 as required
- The proof of the pudding is in the eating!!

Maxwell's Equations

- Div[ϵ_0 **E**] = - Div[**P**] + ρ
- Div[μ_0 **H**] = - Div[μ_0 **M**]
- Curl[**E**] = - ∂/∂t (μ_0 **H**) - ∂/∂t (μ_0 **M**)
- Curl[**H**] = ∂/∂t (ϵ_0 **E**) + ∂/∂t (**P**) + **J**
- $\mu_0 = 4\pi \cdot 10^{-7}$ in SI units
- $\epsilon_0 = 10^{-9}/(36\pi)$ in SI units

Drummond's Observations

- Div[ϵ_0 **E**] = - Div[**P**] + ρ
- Div[μ_0 **H**] = - Div[μ_0 **M**]
- Curl[**E**] = - $\partial/\partial t$ (μ_0 **H**) - $\partial/\partial t$ (μ_0 **M**)
- Curl[**H**] = $\partial/\partial t$ (ϵ_0 **E**) + $\partial/\partial t$ (**P**) + **J**
- Maxwell's equations are linear (usually)
- Principle of superposition applies
- Linear Systems Analysis is possible

Toolkit Required

- Vector algebra
- (partial) differential equations
- Complex representation of oscillatory quantities
- Linear Systems Analysis
- Fourier series and Fourier transforms