Lecture 4

A Model for Dielectrics, Metallic Conductors and Plasmas

ME Monochromatic Wave Eqn

- Del2[***H***]] + $\omega^2\mu\epsilon$***H*** + $i\omega\mu\sigma$***H*** = 0
- Del2[***E***]] + $\omega^2\mu\epsilon$***E*** + $i\omega\mu\sigma$***E*** = 0
- $\mu_0 = 4\pi \, . \, 10^{-7}$ in SI units
- $\epsilon_0 = 10^{-9}/(36\pi)$ in SI units
- If we now REDEFINE ϵ as being $\epsilon + i\sigma/\omega$, we can automatically incorporate conducting media in the solutions with a complex dielectric constant
- This implies that the effect of conductance is the the polarisation **P** is still proportional to the field **E**, but there is a phase lag between the field and the polarisation
- We can also look at the same arguments for the spatial differential Del2[] and find that in a charge-free medium Del2[***X***] = $-k^2$ ***X*** where k is the wave vector

Dispersion Relation

- Putting all these components into the wave equation, we find that
- $k^2 = \omega^2 \epsilon \mu$ for a linear, isotropic, charge-free medium
- Remember that ϵ is generally complex and therefore so is k
- If we define refractive index to be complex following ϵ, then
- $k^2 = \omega^2 / c^2 \, . \; n^2$
- This is the “dispersion relationship” and measures how the wave vector k, varies with frequency

n and ϵ

- Most materials are non-magnetic, assume $\mu=\mu_0$
- $\epsilon = \epsilon_r + i\,\epsilon_i = n^2\,\epsilon_0$
- So if $n = n_r + i\,n_i$
- Then $\epsilon_r / \epsilon_0 = n_r^2 - n_i^2$
- And $\epsilon_i / \epsilon_0 = 2\,n_r\,n_i$
- What’s the point?
- We can measure the refractive index very easily
- therefore we can relate the macroscopic n to the microscopic, complex ϵ

Bringing It Back to Reality

- The wave propagates like exp i(**k** . **r** - ωt + ϕ)
- Suppose it propagates along the z axis -> kz
- The wavelength in free space is λ_v
- and the wavevector is k_v
- Look at the spatial part only
- $\exp(ikz) = \exp\{i (n_r + i n_i) k_v z \}$
- $\exp(ikz) = \exp (i n_r k_v z) \quad \exp(- n_i k_v z)$
- First term is phase factor, second is a decay term
- Measure the phase speed - gives n_r (historically n)
- Measure the decay - gives n_i (historically κ)

Electrons In Materials

- In insulators, electrons are bound to atoms/molecules, but can be displaced by electric fields
- In metals, electrons are free and can "drift" through the metals under the influence of electric fields, slowed by "collisions"
- In plasmas, ions are relatively stationary and electrons move relatively freely under the influence of electric fields
- In a fluid, electrons are bound to atoms/molecules and the entire complex can move (align) with the field subject to a "disordering force" (collisions)

A Model of Dielectrics, Metals and Plasmas

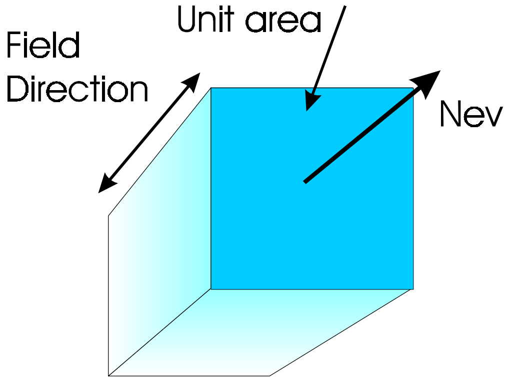
- Assume that in insulators the electrons are attracted to the atom by a restoring force K proportional to displacement - a spring analogy
- Assume that in metals we can approximate the collisions with a drag proportional to velocity - a viscosity analogy
- In addition there is a force on the electron due to the electric field
- Assume there are NO OTHER effects to worry about

Equation of Motion

- $m \, \partial^2\mathbf{r}/\partial t^2 + m\gamma \, \partial\mathbf{r}/\partial t + K\mathbf{r} = -e\mathbf{E}$
- If material is a metal K = 0
- If material is a plasma K,γ = 0
- Assume sinusoidal time dependence, frequency ω
- $- \omega^2 m \, \mathbf{r} - i\omega \, m\gamma \, \mathbf{r} + K\mathbf{r} = -e\mathbf{E}$
- Note that **r** is still proportional to **E**
- $\mathbf{r} = e\mathbf{E} / (\omega^2 m - K + i\omega \, m\gamma)$

Velocity and Conductivity

- Now Let's look at the velocity **v** (diff. **r** wrt t)
- **v** = -e**E** / (mγ+ i(ωm - K/ω))
- Now classically the current density **J** is related to the electric field **E**
- **J** = σ**E** = -Ne**v**
- N is carrier density (Carriers per unit volume)



Conductivity Formula

- So $\sigma = Ne^2 / (m\gamma + i(\omega m - K/\omega))$
- Or $\sigma = Ne^2 (\omega/m) / (\gamma\omega - i(\omega^2 - K/m))$
- Let $K/m = \omega_0^2$, let $Ne^2/(\epsilon_0 m) = \omega_p^2$.
- $\sigma = \epsilon_0 \omega_p^2 \omega / (\gamma\omega + i(\omega_0^2 - \omega^2))$
- IF $\omega_0 = 0$, which implies K = 0 (metal, plasma),
- THEN ω→0 (DC) value is $\sigma(\omega\to 0) = \epsilon_0 \omega_p^2 / \gamma$
- Otherwise $\sigma(\omega\to 0) = 0$
- Enables me to "evaluate" ω_p^2 / γ

Connection to MEs

- We redefined $\epsilon + i\sigma/\omega$
- We said that nothing was happening except the electron motion
- $\epsilon = \epsilon_0 + \epsilon_0 \omega_p^2 / ((\omega_0^2 - \omega^2) - i\gamma\omega)$
- $\epsilon/\epsilon_0 = 1 + \omega_p^2 / ((\omega_0^2 - \omega^2) - i\gamma\omega) = n^2$
- So we can at least notionally compute the refractive index on this basis for three cases:
- Insulators, metals and plasmas