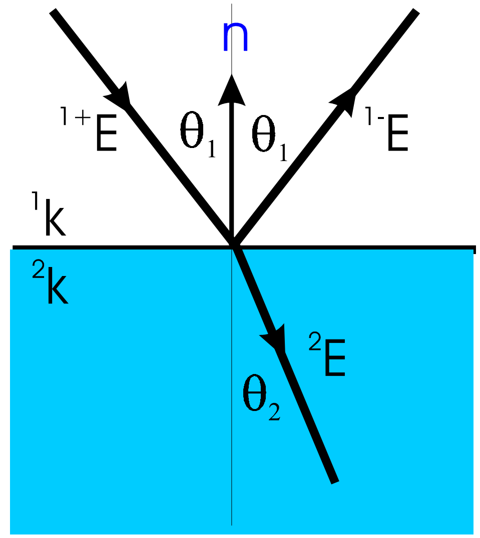
Lecture 10

Reflection at Dielectric and Conducting Interfaces

Boundary Conditions



Boundary Conditions

- Boundary conditions at the interface
- Boundary conditions for dielectrics
- Tangential E and H fields are continuous
  - $({}^{1+}\mathbf{E} + {}^{1-}\mathbf{E}) \times \mathbf{n} = {}^{2}\mathbf{E} \times \mathbf{n}$
  - $({}^{1+}\mathbf{H} + {}^{1-}\mathbf{H}) \times \mathbf{n} = {}^{2}\mathbf{H} \times \mathbf{n}$
  - $({}^{1+}\mathbf{k} \times {}^{1+}\mathbf{E}) \times \mathbf{n} + ({}^{1-}\mathbf{k} \times {}^{1-}\mathbf{E}) \times \mathbf{n} = ({}^{2}\mathbf{k} \times {}^{2}\mathbf{E}) \times \mathbf{n}$
- 1^{st} Case - Electric field parallel to interface
  - transverse electric (TE)
  - s-polarised
- 2^{nd} Case - Magnetic field parallel to interface
  - transverse magnetic field
  - p-polarised - (magnets have poles)

Summary for a TE wave

- For a dielectric interface there is no phase change (0,π)
  - ${}^{1-}E_0 / {}^{1+}E_0 = \sin(\theta_2 - \theta_1)/\sin(\theta_2 + \theta_1)$
  - ${}^{2}E_0 / {}^{1+}E_0 = 2 \cos\theta_1 \sin\theta_2/\sin(\theta_2 + \theta_1)$
- At normal incidence
  - ${}^{1-}E_0 / {}^{1+}E_0 \rightarrow (\theta_2 - \theta_1)/(\theta_2 + \theta_1) \rightarrow (n_1 - n_2)/(n_2 + n_1)$
  - ${}^{2}E_0 / {}^{1+}E_0 \rightarrow 2\, \theta_2/(\theta_2 + \theta_1) \rightarrow 2n_1/(n_2 + n_1)$
- The reflected power ratio
  - $R = \sin^2(\theta_2 - \theta_1)/\sin^2(\theta_2 + \theta_1)$
- The trasmitted power ratio
  - $T = \sin 2\theta_2 \sin 2\theta_1 / \sin^2(\theta_2 + \theta_1)$
- At normal incidence
  - $R = (n_2 - n_1)^2/(n_2 + n_1)^2$, $T = 4n_1n_2/(n_2 + n_1)^2$

Summary for a TM wave

- For a dielectric interface there is no phase change (0,π)
  - ${}^{1-}E_0 / {}^{1+}E_0 = \tan(\theta_1 - \theta_2)/\tan(\theta_2 + \theta_1)$
  - ${}^{2}E_0 / {}^{1+}E_0 = 2 \cos\theta_1 \sin\theta_2/\sin(\theta_2 + \theta_1)/\cos(\theta_1 - \theta_2)$
- At normal incidence
  - ${}^{1-}E_0 / {}^{1+}E_0 \rightarrow (\theta_1 - \theta_2)/(\theta_2 + \theta_1) \rightarrow (n_2 - n_1)/(n_2 + n_1)$
  - ${}^{2}E_0 / {}^{1+}E_0 \rightarrow 2\, \theta_1\, \theta_2/(\theta_2 + \theta_1) \rightarrow 2n_1/(n_2 + n_1)$
- The reflected power ratio
  - $R = \tan^2(\theta_1 - \theta_2)/\tan^2(\theta_2 + \theta_1)$
- The trasmitted power ratio
  - $T = \sin 2\theta_2 \sin 2\theta_1 / \sin^2(\theta_2 + \theta_1)$
- At normal incidence
  - $R = (n_1 - n_2)^2/(n_2 + n_1)^2$, $T = 4n_1n_2/(n_2 + n_1)^2$

Critical Angle

- $n_1 \sin\theta_1 = n_2 \sin\theta_2$ - Snell's Law (n sinθ is conserved)
- If $(n_1/n_2)\sin\theta_1 > 1$ then θ_2 is not real
- The Critical Angle is the special value of incident light that makes $\theta_2=\pi/2$ for a normally incident beam
- We define $\sin\theta_c=n_2/n_1$ for a normal incidence
- Must be moving from high refractive index to low
  - $n_1 > n_2$
- There is no transmitted ray
  - reflection must be perfect!!
- Is there no transmitted ray?
  - What about the fields at the discontinuity?

Evanescent TE Waves

- What's $\cos\theta_2$?
- Snell's law: $\sin\theta_2=(n_1/n_2) \sin\theta_1 = \sin\theta_1/\sin\theta_c$
- $\cos^2\theta_2 = (1 - (n_1/n_2)^2\sin^2\theta_1)= (1 - \sin^2\theta_1/\sin^2\theta_c)$
  - $\cos\theta_2$ is imaginary
  - let $\beta^2 = {}^2k^2 (\sin^2\theta_1/\sin^2\theta_c - 1) = - {}^2k^2 \cos^2\theta_2$
- The transmitted wave propagates spatially as exp(i **${}^2\mathbf{k}$. r**)
- $\exp(i (x\, {}^2k \sin\theta_2 + z\, {}^2k \cos\theta_2)$
- $\exp(-\beta z) \exp(i (x\, {}^1k \sin\theta_1))$
- Wave decays on space scale 1/β
- for glass n_2 =1.5 interface with air n_1 = 1
  - $\theta_c = \sin^{-1}(2/3) = 41.8^o$
  - at 45^o , 1/p about $\lambda_0/2$
- Evanescent wave is the wave that penetrates the second medium decaying as 1/β

Evanescent TE Waves - Phase Changes

- Look at boundary conditions
  - ${}^{1+}E_0 + {}^{1-}E_0 = {}^{2}E_0$
  - ${}^{1+}E_0\, {}^{1+}k \cos\theta_1 - {}^{1-}E_0\, {}^{1-}k \cos\theta_1 = {}^{2}E_0\, {}^{2}k \cos\theta_2$
  - $\beta = i\, {}^2k \cos\theta_2$, $\alpha = {}^1k \cos\theta_1$
- Solve for ${}^{1-}E_0$,${}^{2}E_0$
  - ${}^{2}E_0 / {}^{1+}E_0 = 2\beta/(\alpha - i\beta)$
  - ${}^{1-}E_0 / {}^{1+}E_0 = (\beta + iq)/(\alpha - i\beta)$
  - There are phase changes in the reflected beam
- Look at R, T
  - T = 0 because there is no z component in transmitted wave
  - $R = |{}^{1-}E_0 / {}^{1+}E_0|^2 = |({}^{1-}E_0 / {}^{1+}E_0)({}^{1-}E_0 / {}^{1+}E_0)^*| = 1$
- Total internal reflection as all beam energy is reflected
- Occurs when $\theta_1 > \theta_c$

Frustrated Total Internal Reflection FTIR

- Have shown that field penetrates less dense medium during TIR (to a small extent)
- Consider an air/glass interface and an incident beam: $\theta_1 > \theta_c$
- Imagine that another piece of glass is pressed against the reflective surface
- What happens?

- The effect is remarkably similar to barrier penetration or quantum tunneling
- Beam splitters
- Particularly applicable in microwave where there is 10^5 times the penetration (do the calcs!)