Lecture 12

True Lies - What Happened to B and D

The Fundamental Equations of Electrostatics

- The **F** between two charges q_1 and q_2 separated by a distance r is given by...
  - $\mathbf{F} = 1/(4\pi\epsilon_0)\ (q_1 q_2 / r^2)\ \mathbf{r}$
  - **r** is a unit vector
- The field **E** is given by
  - $\mathbf{F}(\mathbf{r}) = \mathbf{E}(\mathbf{r})\ q_2$
  - By superposition principle force on q_2 is sum of all other forces
  - $\mathbf{E}(\mathbf{r}) = (1/\ 4\pi\epsilon_0)\ \sum_i (q_i / r_i^2)\ \mathbf{r}_i$
  - **E** is directly related to **F**
  - **E** is always the force acting on a unit charge
- In a vacuum we can integrate over a closed surface
  - $\int \mathbf{E}\ .\ \mathbf{dS} = \int \mathrm{div}[\mathbf{E}]\ d\tau = (1/\epsilon_0)\ \int \rho\ d\tau$
  - Flux through Closed Surface <=> Charge Enclosed
  - Consequence of inverse square law nature of **E** (**r**)
  - $\mathrm{div}[\mathbf{E}] = \rho/\epsilon_0$ (in a vacuum) [Gauss]

In a Polarising Medium

- In a dielectric medium there are polarisation effects which act to add a volume dipole moment effect
- Electric field acts to polarise neutral atoms (+ve and -ve charges pulled in different directions)
- If the dipole moment per unit volume is **P**
- It can be shown that the field due to **P** is given by $\mathbf{P}/\epsilon_0$
- Total field is therefore $\mathbf{E} + \mathbf{P}/\epsilon_0$
  - and $\mathrm{Div}[\mathbf{E} + \mathbf{P}/\epsilon_0] = \rho/\epsilon_0$
  - $\mathrm{Div}[\epsilon_0\ \mathbf{E} + \mathbf{P}\ ] = \rho = \mathrm{Div}[\mathbf{D}]$
  - $\mathbf{D} = \epsilon_0\ \mathbf{E} + \mathbf{P}$ - Definition of Displacement **D**
  - ρ is external or free charge

Scalar Potential

Using the vector identity $\mathbf{r} / r^2 = -\mathrm{Grad}[1/r]$
We can write the electric field in terms of a scalar potential
E() = - Grad[]

Units and Other Issues

- **E** is related to the force (Newtons) - units of $N\ C^{-1}$
- Since $\int \mathbf{D} \cdot d\mathbf{S} = \int Div[\mathbf{D}]\ d\tau = \int \rho\ d\tau$
  - Units of **D** are $C\ m^{-2}$
  - $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$
- Units of permittivity ϵ_0 are $C^2\ m^{-2}\ N^{-1}$
- Potential is related to **E** by $\mathbf{E} = -Grad[\Phi]$
- Units of **E** are also $V\ m^{-1} \equiv C\ m^{-2}$

The Approximations - Where We Came In

- $\mathbf{P} = \chi\ \epsilon_0\ \mathbf{E}$ for a many materials the polarisation linear with the applied field
- But not for
  - pyroelectrics
  - piezoelectrics
  - ferroelectrics
  - All these generate a polarisation on application of heat, stress, etc

The Approximations - Where We Came In

- If material is isotropic then $\mathbf{P} = \chi\ \epsilon_0\ \mathbf{E}$
  - and we write $\mathbf{D} = \epsilon\ \mathbf{E}$
  - Note that sometimes the definition is
  - $\mathbf{D} = \epsilon\ \epsilon_0\ \mathbf{E}$ which makes ϵ dimensionless
- $Div[\mathbf{D}] = \rho = Div[\epsilon\ \mathbf{E}]$ in this approximation
- ALWAYS $Div[\mathbf{D}] = Div[\mathbf{P} + \epsilon_0\ \mathbf{E}] = \rho$
- ISOTROPY $Div[\mathbf{E}] = \rho / \epsilon$ (or $= \rho / \epsilon_0\ \epsilon$)
- VACUUM $Div[\mathbf{E}] = \rho / \epsilon_0$

The Fundamental Equations of Electromagnetics

- The **F** between two current elements $I_1 \mathbf{ds_1}$ and $I_2 \mathbf{ds_2}$ separated by a distance r is given by ($\boldsymbol{\delta s} \equiv \mathbf{n}\ \delta s$)
  - $\mathbf{dF} = \mu_0/(4\pi)\ (I_1 I_2 / r^2)\ \{\mathbf{ds_2} \times (\mathbf{ds_1} \times \mathbf{r})\}$
- The induction **B** is given by
  - $\mathbf{dF} = I_2 (\mathbf{ds_2} \times \mathbf{dB_1})$
  - **B** is directly related to **F**
  - **B** is always the force acting on a unit current element
- In a vacuum we can integrate over a closed surface (Stokes Theorem)
  - $\oint \mathbf{B} \cdot \mathbf{ds} = \int Curl[\mathbf{B}] \cdot \mathbf{dS} = \mu_0 \int \mathbf{J} \cdot \mathbf{dS}$
  - $I = \int \mathbf{J} \cdot \mathbf{dS}$
  - $Curl[\mathbf{B}] = \mu_0\ \mathbf{J}$ (in a vacuum)
  - Confirmed by experiment down to atomic scale

In a Magnetising Medium

- In a magnetic medium there are magnetisation effects which add a volume magnetisation induction
- If the magnetisation per unit volume is **M**
- Can be shown that induction due to **M** is given by $-\mu_0$ **M**
  - Compute currents due to volume element of **M**
- Total field is therefore **B** - μ_0 **M**
  - and Curl[**B** - μ_0 **M**] = μ_0 **J**
  - Curl[**B** /μ_0 - **M**] = **J** = Curl[**H**]
  - **B** /μ_0 - **M** = **H** - Definition of Field **H**

Units and Other Issues

- **B** is related to the force (Newtons) - units of N $A^{-1}m^{-1}$
- Since $\int$**H** . **ds** = $\int$Curl[**H**] . **dS** = $\int$**J** . **dS** = I
  - Units of **H** are A m^{-1}
- Units of μ_0 are N A^{-2} [kg m C^{-2}]
- Remember that A = C sec^{-1}

The Approximations - Where We Came In

- **M** = χ **H** and χ is scalar for a many materials
- But not for
  - Ferromagnetics
  - Materials with hysteresis

The Approximations - Where We Came In

- If material is isotropic then **M** = χ **H**
  - and we write **B** = μ **H**
  - Note that sometimes the definition is
  - **B** = μ μ_0 **H** which makes μ dimensionless
- Curl[**H**] = **J** = Curl[**B**/μ_0] in this approximation
- ALWAYS Curl[**H**] = Curl[**M** - **B**/μ_0] = **J**
- ISOTROPY Curl[**B**] = μ**J** (or = μ μ_0**J**)
- VACUUM Curl[**B**] = μ_0**J**

Why We Do What We Do?

- Maxwell's equations would involve six quantities
  - **B**, **H**, **E**, **D**, **J**, ρ
  - But two pairs are proportional in our context
- Simplify and choose the items which make life easiest
  - **H**, **E**, **J**, ρ
  - Slightly odd since only one (**E**) is directly related to force for example