Lecture 15

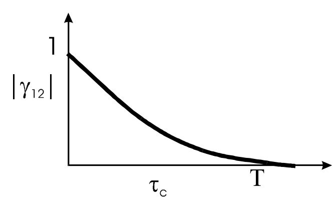
Interference and Interferometers

Interference

- Interference can only occur if the two beams have the same frequency and therefore a defined phase
- If two beams are represented by
  - $\mathbf{E_1} = E_{01} \exp(i\mathbf{k_1}.\mathbf{r} - i\omega t)$
  - $\mathbf{E_2} = E_{02} \exp(i\mathbf{k_2}.\mathbf{r} - i\omega t + i\delta)$
- Then the total energy is
  - $I = |E_t|^2 = \mathbf{E_t}.\mathbf{E_t^*} = |E_{01}|^2 + |E_{02}|^2 + 2E_{01}E_{02}\cos\phi$
  - where $\phi = (\mathbf{k_1} - \mathbf{k_2}).\mathbf{r} - \delta$
- The last term is the interference term and says that the total intensity is not necessarily the sum of the individual intensities and in fact may be larger or smaller

Correlation Function

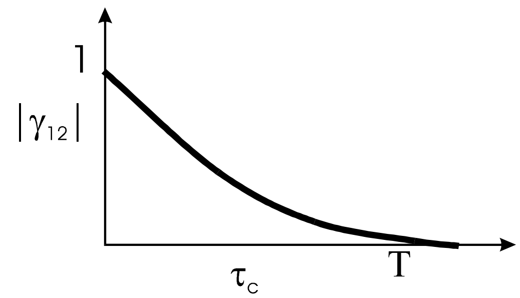
- If the two waves originate from the same source - the interference is in time
- $\Gamma_{12}(\tau) = \langle E_1^*(t).E_2(t+\tau)\rangle$
  - defines the "mutual coherence"
  - or the "correlation function"
  - $\Gamma_{11}(0) \propto I_1$
  - $\Gamma_{11}(\infty) \rightarrow 0$



- Normalised coherence function
  - $\gamma_{12}(\tau) = \Gamma_{12}(\tau)/\sqrt{(\Gamma_{11}(0)\Gamma_{22}(0))}$
  - Time averaged intensity = $I_1 + I_2 + 2\sqrt{(I_1 I_2)}\,\mathrm{Re}[\gamma_{12}(\tau)]$
  - $|\gamma_{12}(\tau)| = 1$ complete coherence
  - $0 < |\gamma_{12}(\tau)| < 1$ partial coherence
  - $|\gamma_{12}(\tau)| = 0$ complete incoherence

Correlation Function

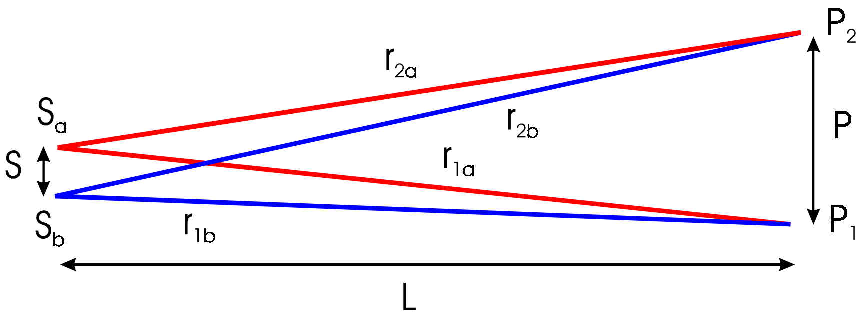
- Correlation decays with characteristic time τ_c
- Form of $|\gamma_{12}(\tau)|$ is often like an exponential decay



- Auto-correlation function (also known as self-correlation function) is linked to power spectrum $P(\omega)$ through fourier transform - the Wiener-Khintchine Theorem
- $\mathscr{F}(\Gamma_{11}(\tau)) = P(\omega)d\omega$

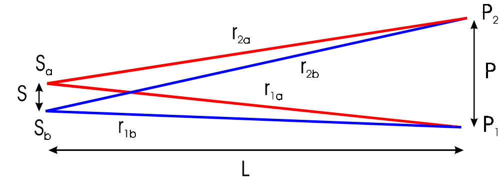
Spatial and Temporal Correlation

- Temporal coherence
  - Time is the only variable
  - Length is only used along the direction of propagation
- Spatial coherence
  - Links two space points, not on DOP
  - One point is generally fixed.
- Consider the following situation



Spatial and Temporal Correlation

- Spatial coherence between two points is defined as:

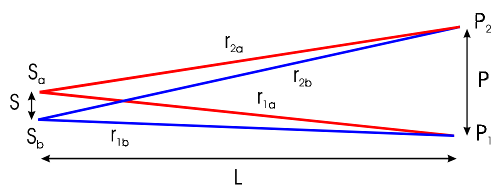


- $$\gamma_{12}(\tau) = \frac{<E_1^*(t)\ E_2(t+\tau)>}{\sqrt{I_1 I_2}}$$

Each point has two components to E, BUT the sources are mutually incoherent so the cross-terms disappear in the average

$$\gamma_{12}(\tau) = \frac{<E_{1a}^*(t)\ E_{2a}(t+\tau)>}{\sqrt{I_1 I_2}} + \frac{<E_{1b}^*(t)\ E_{2b}(t+\tau)>}{\sqrt{I_1 I_2}}$$

Spatial and Temporal Correlation



- If the sources are point, monochromatic, then each term is almost a self-coherence function
- Common form for a self-coherence function is:

$$\gamma(\tau) = \exp(-i\omega\tau)\ \exp(-|\tau|/\tau_0)$$

- where the τ is just the path difference converted to a time $\tau_x = (r_{xa}-r_{xb})/c$
- If we assume vertical distances >> horizontal we find that $\tau_a - \tau_b = SP/(cL)$
- Write out the full coherence function as:

$$\exp(-i\omega\tau_a)\ \exp(-|\tau_a|/\tau_0)$$
$$(1 + \exp(-i\omega(\tau_b - \tau_a))\ \exp(-(|\tau_b| - |\tau_a|)/\tau_0)/2$$

Spatial and Temporal Correlation

- Taking the modulus and making the appropriate substitutions

$$|\gamma_{12}(\tau)| = \frac{1 + \cos\left(\frac{\omega SP}{cL}\right)}{2} \exp\left(\frac{|\tau|}{\tau_0}\right)$$

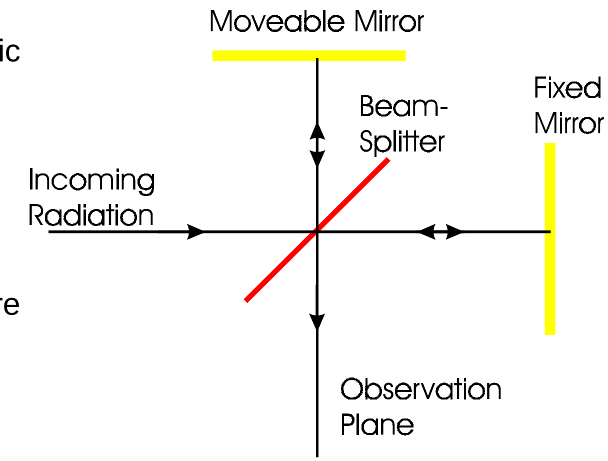
- which means that the oscillations can be related to the source separation S and the distance to object L
- Know one - get the other!

Interferometers

- Used for measuring distance and/or spectral response
- Two major types:
  - Michelson
  - Fabry-Perot
- Michelson Interferometer
  - Uses amplitude division of the wavefront
  - recombines wavefront after two different paths
  - Produces the fourier transform of the source power
  - Therefore also known as “fourier transform interferometer”
  - Need to do FT - need a computer

Fourier Transform Interferometer

- Monochromatic source has fringes every $\lambda/2$ of mirror displacement
- All sources have a peak when paths are equal - “zero path difference” - ZPD



- Maximum path difference depends upon design - movement of 1000mm is possible - 2m path difference
- Signal at the detector is $I_1 + I_2 + 2\mathrm{Re}(\Gamma_{12}(\tau))$
- $\mathscr{F}(\Gamma_{12}(\tau)) = P(\omega)d\omega$ (W-K Theorem)

FTI Resolving Power

- Consider two closely spaced monchromatic frequencies
  - can be resolved if fringe frequencies can be distinguished
  - of the order of 1 fringe different at maximum displacement
- So if D is total distance moved then

$$D = \frac{M\lambda}{2} = \frac{(M+1)\ (\lambda-\Delta\lambda)}{2}$$

- Solving this for large M gives

$$\text{resolving power} = \frac{\lambda}{\Delta\lambda} = M = \frac{2D}{\lambda}$$

- for D = 1000mm and λ = 1µm the resolving power is about 10^6
- For a grating spectrometer the resolving power is more like 10^4
- For a prism spectrometer more like 10^2

FTI Frequency Coverage

- The highest fringe frequency corresponds to the shortest wavelength that can be observed
  - need at least two samples
- If step size is d. Then minimum wavelength is d/4
- BUT there is a problem with aliasing - frequencies become confused.
- Other things frequently restrict the wavelength region as well as the step spacing - deliberately.