PHY 353S - Electromagnetic Waves
Spring 2000
Problem Set 1 - due 5:00pm February, 10th

You may use any method - except cheating - to solve these problems
There are no trick questions here.
Some may require thought, but none - so far as I know - require magic for the solution

1. An electromagnetic wave is defined (in SI units) by the following function:

$$\vec{E} = (-3\hat{i} + 3\sqrt{3}\,\hat{j})$$
$$(10^4)\exp(i[(1/3)(\sqrt{5}x + 2y)\pi \times 10^7 - 9.42 \times 10^{15} t])\ \mathrm{V/m}$$

Find:

i) The direction along which the electric field oscillates
ii) The scalar value of the amplitude of the electric field
iii) The direction of propagation of the wave
iv) The propagation wavevector and wavelength
v) The frequency and angular frequenct
vi) The speed

2. The average value of the Poynting vector for sunlight arriving at the top of the earth's atmosphere (1.5×10^{11}m from the Sun) is about $1.37\mathrm{kW/m^2}$

i) Compute the average radiation pressure exerted on a metal reflector facing the Sun
ii) Approximate the average radiation pressure at the surface of the Sun whose diameter is 1.4×10^9m

3. Augustin Louis Cauchy (1789-1857) came up with an empirical formula for the refractive index of materials as:

$$n = C_1 + C_2/\lambda^2 + C_4/\lambda^4 + \ldots$$

i) Can you explain the significance of C_1?
ii) Crystal quartz has refractive indices of 1.557 at 410nm and 1.547 at 550nm. Using a two-term Cauchy expansion, calculate the index of refraction of quartz at 610nm.
iii) Why should you never try to do the calculation above?

4. An astronaut of mass 100kg is stranded in interstellar space with only a 10W flashlight with an inexhaustible power supply. The flashlight may be considered to be unidirectional. How long will it take for the astronaut to reach a velocity of $10\mathrm{ms^{-1}}$ using the power of the flashlight?

5. A parabolic radar antenna with a 2m dish transmits 200kW pulses of energy. If the repetition rate is 500 pulses per second and each pulse lasts 2μS, calculate the average reaction force on the antenna. Do we need to worry about this in antenna design?

6. An ideal linear polariser is rotated at an angular rate ω between two ideal linear polarisers at 90° to each other. Show that the emergent flux density *I* will be modulated according to:

$$I = (I_1/8)(1 - \cos 4\omega t)$$

where I_1 is the incident flux density.

7. Two incoherent light beams represented by (1,1,0,0) and (3,0,0,3) are superimposed

i) Describe in detail the polarisation of the two beams
ii) Determine the resulting Stokes parameters of the combined beam and describe its polarisation state
iii) What is its degree of polarisation?

8. The refractive indices for carbon disulphide at 490nm and 620nm are 1.65338 and 1.62425 respectively. Assuming Cauchy's equation (see 3 above) calculate the phase and group velocities of light in carbon disulphide at the mean

wavelength of 555nm. Compare your results with Michelson's experimental results of 1/1.758 of the velocity of light in a vacuum for white light and 1.4% faster for "orange-red" light than for "greenish-blue". (Michelson wasn't entirely right!)

9. The "skin depth" has been discussed as the depth to which current penetrates from the surface of a metallic conductor. If the skin depth has to be the thickness of at least one atom. What is the highest frequency that can be transmitted through a copper wire?

10. The ratio of E/H for an electromagnetic wave has the dimensions of a resistance or impedance.
    i) Calculate the impedance of a vacuum (the "impedance of free space") Z_0, in units of Ohms.
    ii) What is the relationship between the impedance of a medium of refractive index n (assumed real) and Z_0?