PHY 353S - Electromagnetic Waves
March 1998 - Problem Set 2

Due 5:00pm Tuesday, March 30th, 1999

(You may use any method apart from cheating to solve these problems)

1. The plates of a Fabry-Perot interferometer have a reflectivity of 0.85. It is designed to be able to resolve the components of the H_α line of hydrogen (656.279nm) which is a doublet of wavelength difference 0.0132nm
   a. Calculate the minimum separation of the plates required in order to achieve this.
   b. If, instead of having separate mirrors the Fabry-Perot interferometer is made by coating both ends of a plane parallel "chunk" of glass of refractive index 1.35, calculate the minimum thickness of the glass required.
   c. Estimate the flatness of the optical surfaces required in order to perform the above experiment successfully.

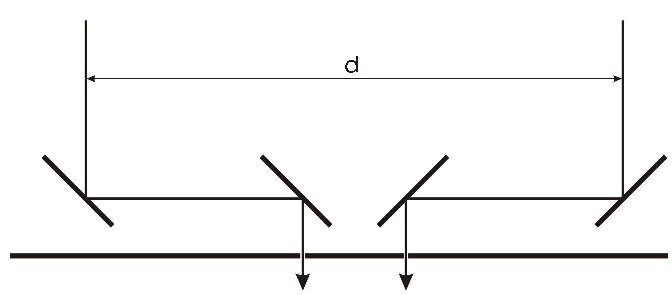
2. Optical fringes are frequently used to compare the lengths of objects. Consider the following situation: Two steel rods 2.54cm in diameter are to be compared in length. They are placed upright on an optical flat with their centres 5cm apart. A second optical flat is placed across the top and illuminated from above with a He-Ne laser at 632.8nm. It is found that there is a fringe pattern on the top of each rod.
   a. Draw a simple ray diagram of this situation
   b. Determine the formula relating the fringe spacing to the difference in length of the rods
   c. If the fringe pattern has 5 fringes/cm what is the difference in length of the rods?
   d. What is the minimum difference in length which could be determined using this method?

3. After the last problem set there was some discussion about the difference between a plane polarised beam of radiation which is the sort of thing discussed in our solution of Maxwell's equations, and what you get if you pass "natural light" through a plane polariser.
   a. Discuss the similarities and differences between the two cases
   b. How would you devise an experiment to accurately tell the difference between the two?

4. We are going to construct a simple telescope for observing the planets and stars
   a. Show how the Airy pattern in the observation plane is related to the angular deviation of the incoming light from the telescope axis and the size of the telescope aperture.
   b. If we want to be able to observe the rings of the planet Saturn, determine a reasonable minimum size of telescope aperture.

5. The pupil of the eye has an average diameter of 2.5mm in daylight and about 8mm at low light levels.
   a. At what distance in daylight and low light should you be able to resolve two orange dots (λ about 600nm) on a piece of paper, assuming that your eye is diffraction limited?
   b. By experiment, what is the distance?
   c. Comment on the perfection of the optics of your eyeball.

6. In a classical Young's slits experiment, we always assume that the wave striking the slits is a plane wave, but that isn't the case.
   a. If the circular source for a Young's slits experiment subtends a small finite angle, α, at what angle (approximately) do the fringes disappear as a function of the slit spacing, d, and the slit size, a?
   b. Michelson exploited this effect for measuring the angular size of stars. He mounted his slits on the front of a 100inch telescope and ingeneously expanded the slit separation as shown in the diagram below. He found that when the mirrors were 27ft apart, that the fringes of the visible image of the star Arcturus disappeared. What was the angular size of the star?
   c. If the star Arcturus is about 3300AU away, what is it's diameter as compared to our sun?



7.

a. If an optical system has an ABCD matrix which is the identity or unity matrix, show or argue that this optical system has no effect on incoming rays - except to relay them through the displacement of the optical system - ie if the optical system is a mile long, then they have been moved a mile without other effect.

b. A "lens relay" is formed by an infinite sequence of thin lenses of focal length f, separated from each other by a distance L. Show by ABCD matrix methods that for a particular relationship between f and L the effect of this optical system after 2 lenses is the negative of the identity matrix and after 4 lenses is the identity matrix itself.

c. What is the effect of 4n lenses where n is a large integer?

d. A real optical system which has no effect on the input ray is evidently a nonsense in reality. What other effects might have to be accounted for in a real system?

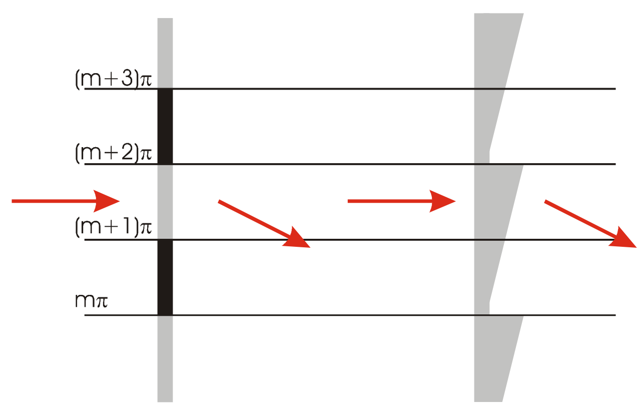
8. I am observing the stars through a small (20cm) telescope in my backyard. I assume that the radiation reaching me from the star is plane parallel. If a person walks in front of the telescope, then I don't see anything, but if a satellite (which is larger) passes through the field, it isn't obscured.

a. Why not?

b. Show that if the relationship between the size of the telescope aperture, d, the size of the object, D, its distance, a, and the wavelength of light λ is such that $\lambda a/(dD) >> 1$, then the object doesn't obscure the telescope.

c. But this predicts that an enormous object doesn't obscure the star, so long as it is far enough away. What are the limitations of this formula?

9. We are attempting to make a Fresnel lens (page 144-145 of the notes). A section of the zone plate, far from the optical axis, is shown in the left-hand part of the diagram. The zone plate consists of alternating areas of opaque and transparent material (in the other plane, they are circles). The edges of the zone are points at which the phase of a plane perpendicular wave, brought to a focus a distance f away, changes phase by a factor of π.



a. Show that the radius of the edge of the zone, m, is given by $r = \sqrt{(m\lambda f)}$ for large m and f

b. Show that for large m and f that the clear or opaque area of a zone is a constant, ie apart from angular factors, that the light through each zone out to infinity is a constant (be grateful for the angular factors!)

c. In an attempt to make a "zone plate" more efficient, the profile in the right-hand side of the diagram is adopted. The idea is that the wedges form a phase delay of 2π from end to end and thus keep a constant phase at the focus. The deviation of the ray by the prism is being ignored. Show that the formula for the wedge angle, α, as a function of the refractive index of the material, n, is $\alpha = r/((n-1)f)$.

10. Forgetting all this Fresnel nonsense in the last question for the minute, let's focus on the question of making a lens from a series of circular prisms.

a. If we assume that the light in the above example is refracted, but not diffracted, what is the relationship for the wedge angle, β, as a function of the radius, r?

b. Since the formula for the wedge angle, β, is not the same as the formula for the wedge angle, α, in the previous question, can you reconcile these and find the right formula for the wedge angle?