Mechanics Module 6
Student Guide

Concepts of this Module

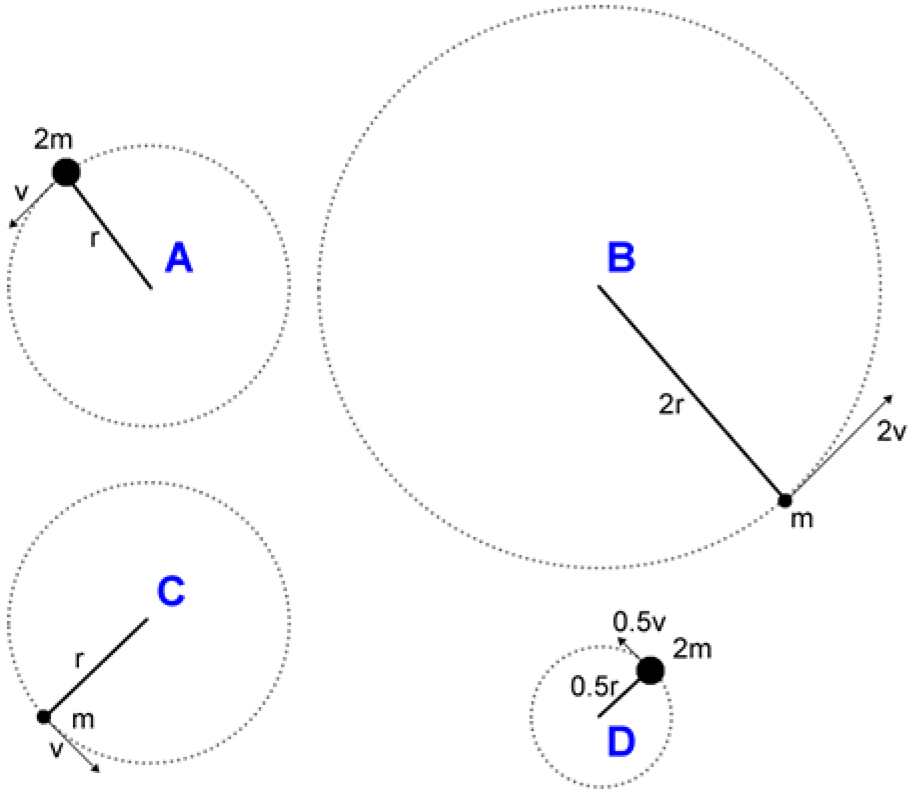
- Angular Momentum
- Rotational Motion
- Torque
- Moment of Inertia
- Rotational Dynamics

Course Concepts *Activity 1*

The figure on the next page shows four cases of point particles in uniform circular motion: A, B, C, and D. The motion of all four masses are in the horizontal plane. Recall that for a point particle of mass *m* moving with speed *v* in a circle of radius *r* the angular momentum:

$$L = mvr$$

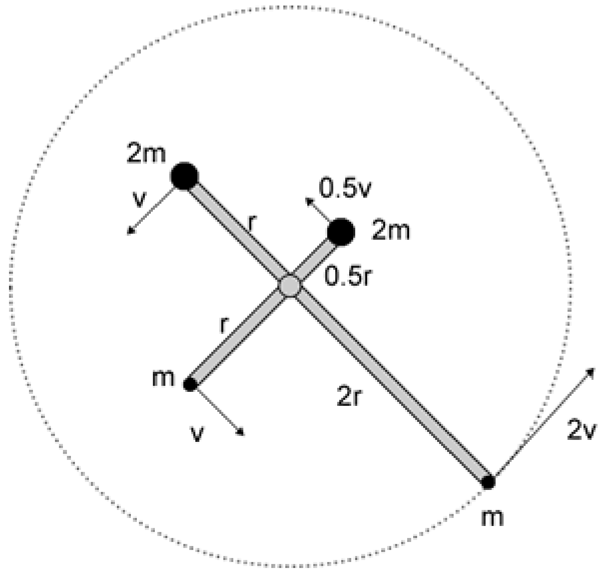
A. Rank in order from the largest to the smallest the angular momentum L of the four cases. Explain your reasoning.
B. Recall that for an object in circular motion with radius r and speed v, the angular velocity ω is defined as: $\omega \equiv \frac{v}{r}$. Call the angular velocity of case A ω_A. Express the angular velocity of cases B, C, and D in terms of ω_A.
C. Now write the angular momentum of the four cases in terms of m, r, and ω_A. Does this form make it easier or more difficult to rank the angular momenta as in Part A? Explain.



Activity 2

The four masses of Activity 1 have been connected to a massless frame that is rotating in the horizontal plane about the central pivot point, as shown on the next page. The "spokes" of the frame are rigid.

A. Will the motions of the masses tend to distort the frame in the horizontal plane?
B. What is the total angular momentum of the four masses?
C. What is the moment of inertia of the combined system of four masses?



Course Concepts *Activity 3*

Whirl the ball on a string in a horizontal circle, being careful not to hit anybody with it. Try to maintain the ball at constant speed. You will find it useful to run the string through the supplied drinking straw and hold the straw in your hand, keeping the string taut with your other hand. If you make the axis of rotation directly over your head you will be much less likely to hit yourself in the head with the ball.

A. Was the linear momentum $\vec{p} = m\vec{v}$ of the ball conserved as the ball moved in uniform circular motion? Explain.
B. Was the kinetic energy ½ mv^2 of the ball conserved as the ball moved in uniform circular motion?
C. Was the angular momentum of the ball, $L = mvr = mr^2\omega$, conserved as the ball moved in uniform circular motion?
D. Whirl the ball again in a horizontal circle. Reduce the radius r of the circle to about ½ r by pulling on the string hanging below your hand. What happened to the speed of the ball? Did you notice anything about the pull on your hand by the string? If yes, what?
E. Was the kinetic energy conserved as the radius of the circle was being reduced? Explain.
F. Was the angular momentum conserved as the radius of the circle was being reduced? Explain.
G. If the speed of the ball was v when the radius of the circle was r, what was the speed when the radius of the circle was ½ r? If the magnitude of the angular velocity of the ball was ω when the radius of the circle was r, what was its magnitude when the radius of the circle was ½ r?

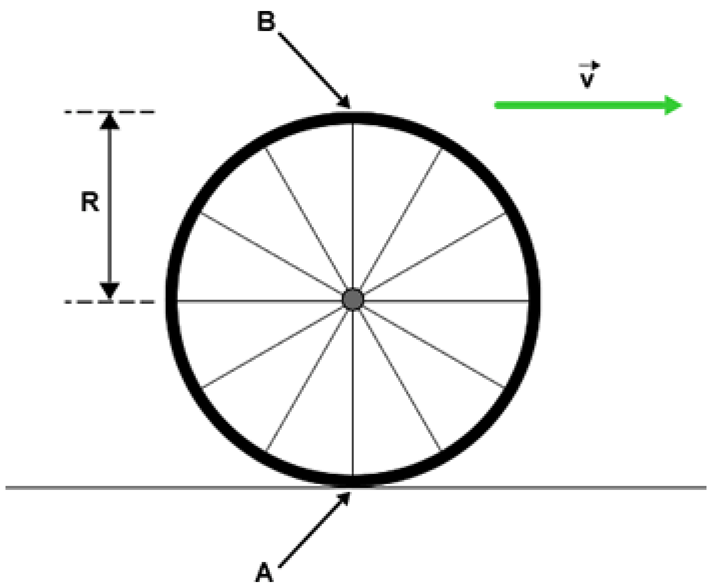
Course Concepts *Activity 4*

A Flash animation illustrating some of the points of this Activity and the next one is available at:

http://www.upscale.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/RollingDisc/RollingDisc.html.

A full screen version which is easier for a group of people to see is at:

http://www.upscale.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/RollingDisc/RollingDisc.swf

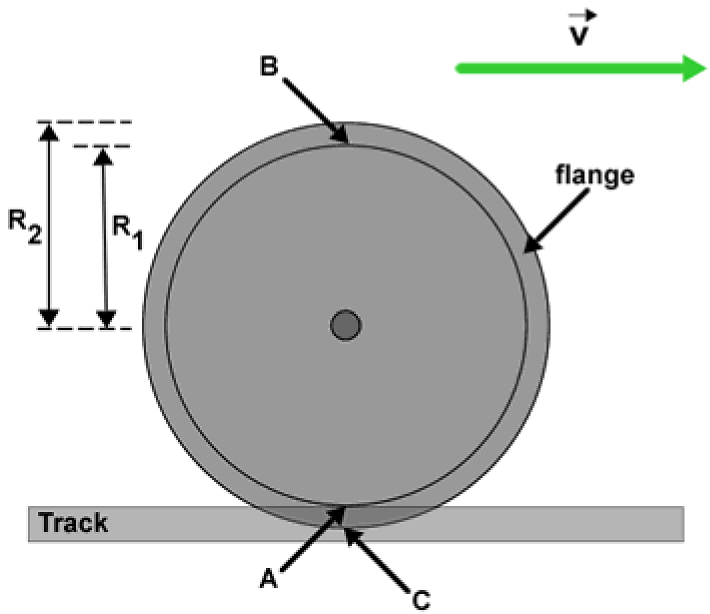


A bicycle wheel of radius **R** rolls to the right without slipping. The velocity of the axle of the wheel relative to an observer standing on the road is $\vec{v}$. At the moment shown in the figure Point **A** is in contact with the road, and Point **B** is at the top of the wheel.

A. At the moment shown what is the instantaneous velocity of Point **A** relative to an observer standing on the road?
B. For the person riding on the bicycle, about what point is the wheel rotating? Why is this point different than any other point on the wheel? What are the velocities of Points **A** and **B** and the axel at the moment shown in the figure? What is the angular velocity ω_B of the wheel?
C. For an observer standing on the road, about what point is the wheel rotating? For this observer what is the angular velocity ω_R of the wheel? What is the instantaneous velocity of Point **B** for this observer at the moment shown in the figure?

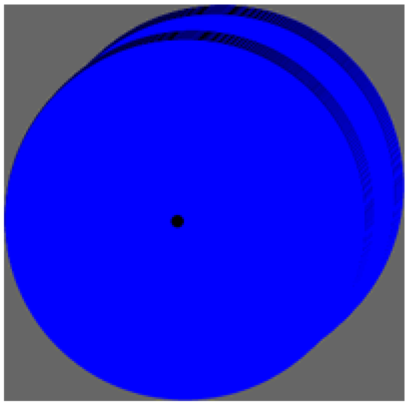
Activity 5

To keep them from slipping off the tracks, train and streetcar wheels have a *flange*, as shown. The radius of the part of the wheel in contact with the rail is $\mathbf{R_1}$, and the radius of the flange is $\mathbf{R_2}$. The wheel is rolling to the right without slipping. The velocity of the axle of the wheel for an observer who is stationary relative to the track is $\vec{v}$. At the moment shown in the figure Point **A** is in contact with the track and Point **C** is at the bottom of the flange. At the moment shown what is the velocity of Point **C** for an observer who is stationary relative to the track? Does this answer surprise you? Explain.

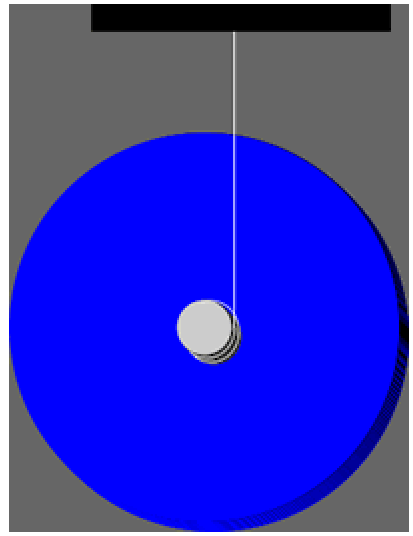


Activity 6

A. Here is a figure of a yoyo that is in free fall: the string is not attached to anything and is not shown in the figure. Draw a free body diagram of the forces acting on the yoyo. Assume that air resistance in negligible.



B. Here is a cross section of a yoyo that is falling with the end of the string fixed to a support. In Part A, you could reasonably assume that the yoyo is a point particle. The free body diagram for this case must treat the yoyo as an extended body, and where the forces are exerted on it is important. Draw an *extended free body diagram* of the forces acting on the yoyo.



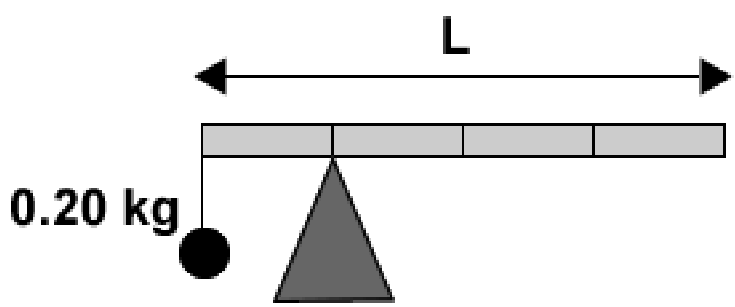
C. If both yoyos are released at the same time from the same height do they both fall at the same rate? Which moves fastest? Confirm your prediction by dropping the yoyo with and without you holding the string; catch the yoyo of Part A so it doesn't get damaged by colliding with the floor or tabletop.

D. Explain the results of Part C qualitatively using Newton's Laws.

E. For the yoyo of Part B, can the force exerted on the yoyo by the string ever do work on it? Explain the result of Part C qualitatively using conservation of energy.

Course Concepts *Activity 7*

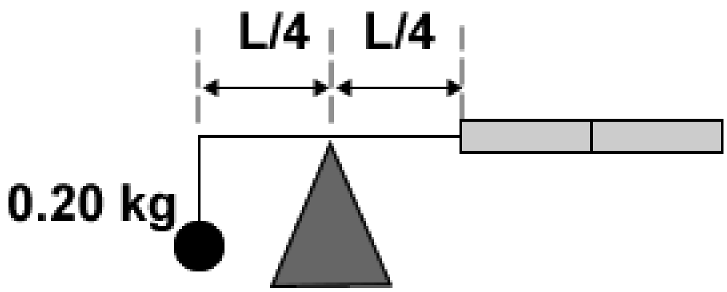
A. A uniform meter stick of length L = 1.0 m has a 0.20 kg mass suspended by a string from its left side, and rests on a pivot that is 0.25 m from the left side. If the meter stick is balanced what is its mass?



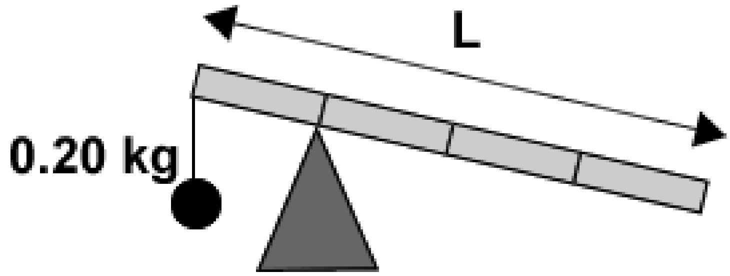
B. What is the force exerted on the meter stick by the pivot?

C. Evaluate the total torque exerted on the meter stick about the pivot point. Repeat for the total torque evaluated about the left side of the meter stick, where the 0.20 kg mass is attached. Repeat for the total torque evaluated about the far right side of the meter stick. Can you generalize these results to a statement about evaluating the torque for a body that is in equilibrium? Explain.

D. The meter stick has been cut in half. The right half is attached to a massless frame that is free to rotate about the pivot. The 0.20 kg mass is suspended from the left side of the frame 0.25 m from the pivot. Are the half meter stick, frame and mass balanced?. What is the mass of the half meter stick? Is your answer consistent with that mass of the full meter stick of Part A? Explain.

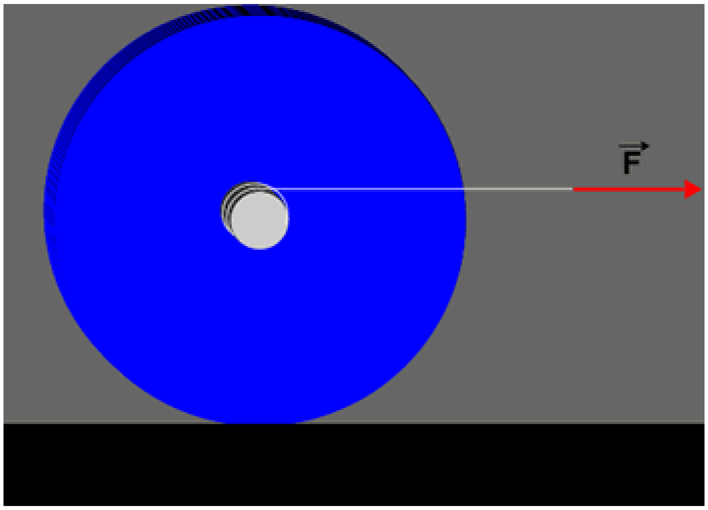


E. The full meter stick and 0.20 kg mass is tilted, held at rest, and gently released. What will be its motion? Explain.

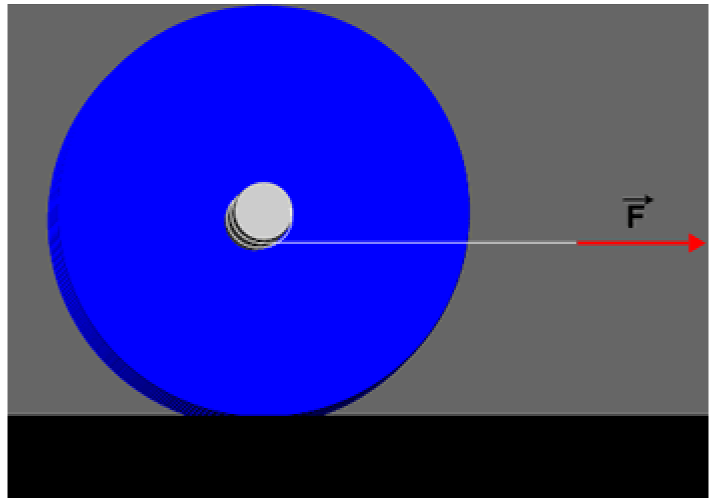


Course Concepts *Activity 8*

A. A yoyo sits on the tabletop, and is gently pulled to the right by the horizontal string, which is wound about the axel as shown in the cross-section view. The pull is gentle enough that the yoyo does not slip. Predict the motion of the yoyo. Using the supplied yoyo, confirm your prediction.



B. A yoyo sits on the tabletop, and is gently pulled to the right by the horizontal string, which is wound about the axel as shown in the cross-section view: note that now the string is attached to the bottom of the axel. The pull is gentle enough that the yoyo does not slip. Predict the motion of the yoyo. Using the supplied yoyo, confirm your prediction.

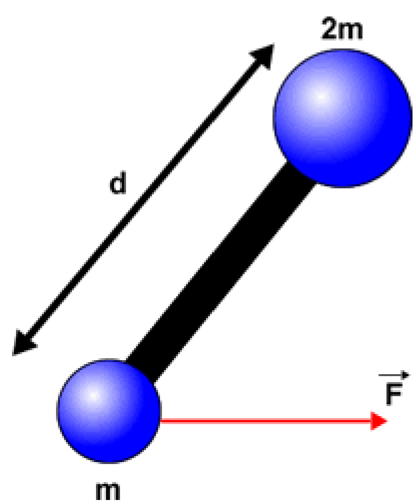


C. Explain the results of Parts A and B. Assume that the radius of the axel of the yoyo is r, and the radius of the yoyo itself is R. What are the total torques acting on the yoyo about its axis of rotation for Parts A and B?

D. Suppose that in the arrangement of Part B the string is not horizontal, but instead pulls the yoyo to the right and up. As the angle of the string is increased predict what will happen. Test your prediction. Can you explain?

Course Concepts Activity 9

A dumbbell consists of two masses m and $2m$ separated by a distance d by a massless rod. The dumbbell rests on a frictionless horizontal table, and a force F is pulling mass m to the right. In the figure we are looking down at the dumbbell from above

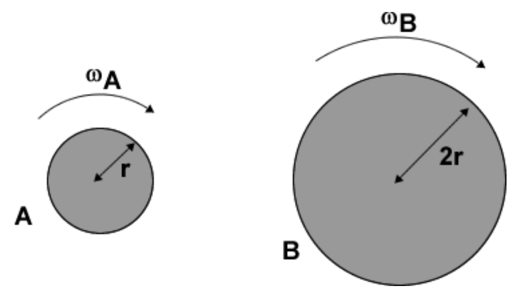


A. Are there one or more forces that can be applied to the dumbbell that will cause it to move with only translational motion, without any rotation? If yes what is/are those forces, magnitude, direction, and applied to what part of the dumbbell? If not, explain.

B. In addition to the force shown in the figure are there one or more forces that can be applied to the dumbbell that will cause it to move with only constant translational speed, without any rotation or acceleration? If yes what is/are those forces, magnitude, direction, and applied to what part of the dumbbell? If not, explain.

Activity 10

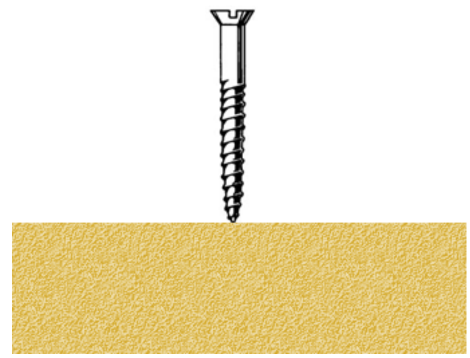
Two uniform cylinders are made of the same material, have the same thickness, and are rotating about their axes of symmetry. Cylinder B has twice the radius of Cylinder A. Recall that the moment of inertia of a cylinder of mass *M* and radius *R* that is rotating about its axis of symmetry is $I = \frac{1}{2}MR^2$.



a) If the two cylinders are to have the same angular momentum, what must be the relation between their angular speeds ω_A and ω_B?
b) If the two cylinders are to have the same rotational kinetic energy, what must be the relation between their angular speeds ω_A and ω_B?

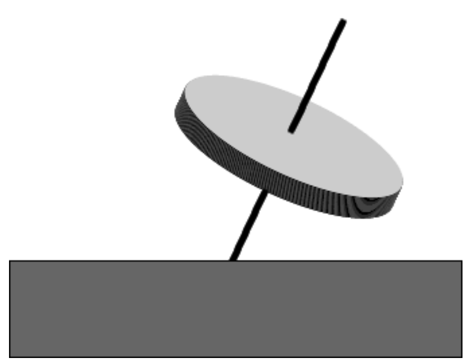
Activity 11

You are driving a screw into a piece of wood. In addition to the screw's rotation it moves down into the wood. What is the relation between the direction of the angular momentum vector of the turning screw and the direction it is moving into the wood?

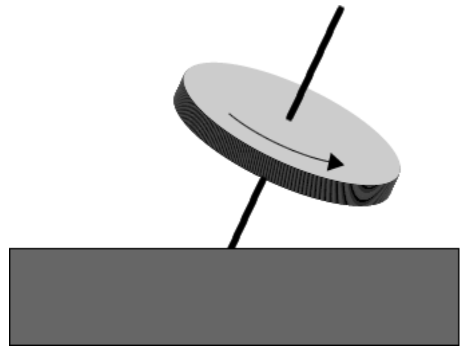


Activity 12

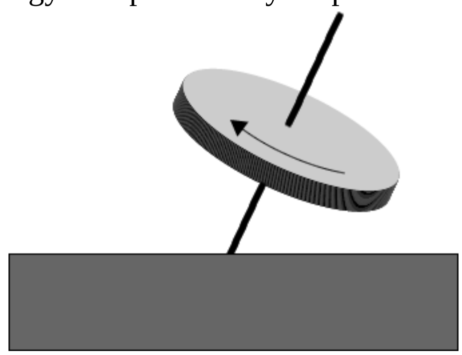
A. Here is a gyroscope which is **not** spinning. What is the direction of the torque exerted on the gyroscope? What will this torque make the gyroscope do?



B. Now the gyroscope is spinning as shown. What is the direction of the torque exerted on the gyroscope?



C. What is the direction of the angular momentum vector of the gyroscope?
D. From your answer to Part B what can you conclude about the magnitude of the *vertical* component of the angular momentum as the motion proceeds?
E. If the spinning gyroscope did what you said in Part A for the gyroscope that wasn't spinning, what would happen to the vertical component of the angular momentum? Is this possible?
F. Now predict the direction of precession of the gyroscope. Check your prediction with the supplied gyroscope.
G. Now the gyroscope is rotating in the opposite direction of Part B. Repeat Parts B – F.



There is a Flash animation of a precessing top at:

http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/Precession/Precession.html

This guide was written in May 2008 by David M. Harrison, Dept. of Physics, Univ. of Toronto

Last revision: October 11, 2012 by Jason Harlow.